Valuing Water Quality Improvements Using Revealed Preference Methods When Corner Solutions Are Present

Daniel J. Phaneuf, Catherine L. Kling, and Joseph A. Herriges

Revealed preference methods for valuing water quality improvements use observed behavior to value water resources indirectly. Particularly when valuing water resources for recreation purposes, it is typical to observe many corner solutions in the data. Corner solutions arise when consumers visit only a subset of the available recreation sites, setting their demand to zero for the remaining sites. In this paper, we set up a general utility theoretical model of recreation choice to use as a benchmark, discussing the implications for empirical specification. Next, we examine three common empirical models, assessing the degree to which they match the theoretical benchmark in their current form and how they might be adapted to better represent the theoretical model. Finally, we estimate each model using data on water-based recreation in the Great Lakes region, providing and contrasting welfare estimates for changes in water quality.

Theoretical Model of Recreation Choice When Corner Solutions Are Present

The model we adopt here has been called the quality differentiated goods model, for which demand for a commodity such as recreation depends on the quality of the commodity as well as prices and income. We will treat this model as an "ideal" that will be used to assess alternative empirical recreation demand models. A consumer faced with a choice of alternative recreation sites to visit solves an optimization problem of the form

$$(1)\quad \max_{x,z} U(\mathbf{x}, z, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$$

$$\text{s.t.}$$

$$\mathbf{p}'\mathbf{x} + z = y, \quad x_j \geq 0, \quad j = 1, \ldots, M$$

where $\mathbf{x}$ is a vector of visits to recreation sites, $\mathbf{p}$ is a vector of site prices, $\mathbf{q}$ is a vector of the corresponding qualities, $\boldsymbol{\epsilon}^{\text{int}}$ is a vector of error terms that generates integer values for each item in the $\mathbf{x}$ vector, z is a *numéraire*, M is the total number of recreation sites, and y denotes income.

The solution to this problem can be characterized by Kuhn-Tucker (KT) conditions. Equivalently, a heuristic approach outlined in Bockstael, Hanemann, and Strand can be used. The advantage of this latter approach is that it clearly demonstrates the implications of the nonnegativity constraints for model structure. The procedure is to decompose the optimization in equation (1) into a series of conditional optimization problems, from which the unconditional optimum is chosen.

To begin with, note that there are 2^M possible combinations of nonzero subsets of the sites; that is, there are 2^M possible "corners" from among which the consumer can choose. For each of these corners, a conditional maximization problem can be specified. The solution to the unconditional problem will be identical to the solution to the conditional problem that yields the greatest utility. Let ω index the 2^M possible combinations of the sites (i.e., $\omega = 1, \ldots, 2^M$) and C_ω denote the combination sets themselves (e.g., $C_1 = \varnothing$, $C_2 = \{1\}$, $C_3 = \{2\}$, etc.). Next, define a conditional maximization problem associated with each C_ω:

Daniel Phaneuf is an assistant professor in the Department of Agricultural and Resource Economics at North Carolina State University. Joseph Herriges and Catherine Kling are associate and full professors, respectively, in the Department of Economics at Iowa State University.

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$$\text{(2)} \quad \max_{x,z} U(\mathbf{x}, z, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$$

$$\text{s.t.} \quad \sum_{j\in C_\omega} p_j x_j + z = y \quad \text{and} \quad x_j = 0, \quad j \in C_\omega.$$

The solution to this problem yields a set of Marshallian demands and an indirect utility function for each corner solution. Denote these functions as $\mathbf{x}_\omega = \mathbf{x}_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$ and $\mathbf{V}_\omega = \mathbf{V}_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$, respectively, where $\mathbf{p}_\omega = \{p_j: j \in C_\omega\}$ is the vector of prices for those commodities that have not been constrained to zero. Note that the conditional demand for x_j depends on its own price and the price of other goods in the constrained choice set ω, but not on the prices of goods constrained to zero. Thus, when the demand for x_j in the demand system is exactly zero, p_j does not enter the conditional demand functions for the remaining $n - 1$ goods. The presence of corner solutions changes the very structure of demand systems!

In general, the nonzero demands are functions of all of the qualities of all goods. However, if weak complementarity exists between visits to sites and their respective qualities, then the constrained demand and indirect utility functions associated with this problem can be simplified to $\mathbf{x}_\omega = \tilde{\mathbf{x}}_\omega(\mathbf{p}_\omega, y, \mathbf{q}_\omega, \boldsymbol{\epsilon}^{\text{int}})$ and $\mathbf{V}_\omega = \tilde{\mathbf{V}}_\omega(\mathbf{p}_\omega, y, \mathbf{q}_\omega, \boldsymbol{\epsilon}^{\text{int}})$. Again, this has important implications for applied researchers. It is inappropriate to include the quality of substitute sites in the conditional demand equations when weakly complementary holds.

The solution to the unconstrained problem can be recovered from the set of constrained problems by noting that the consumer will choose to visit that set of sites that yields the highest utility, i.e., $\mathbf{V}(\mathbf{p}, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}}) = \max_\omega\{\tilde{\mathbf{V}}(\mathbf{p}_\omega, y, \mathbf{q}_\omega, \boldsymbol{\epsilon}^{\text{int}})\}$. Likewise, the unconditional demand equations are given by $\mathbf{x}(\mathbf{p}, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}}) = \Sigma_\omega\, \delta_\omega(\mathbf{p}, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})\mathbf{x}_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$, where $\delta_\omega(\mathbf{p}, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$ equals 1 if $\omega = \arg\max_\tau\{\tilde{\mathbf{V}}(\mathbf{p}_\tau, y, \mathbf{q}_\tau, \boldsymbol{\epsilon}^{\text{int}})\}$ and equals 0 otherwise. This characterization of the consumer's maximization problem as a two-stage process reveals the important properties of the recreation demand decision in a utility theoretic formulation. In assessing empirical models of recreation demand, the features of the theoretical solution can provide guidance regarding specification. An accurately specified empirical model should

- model simultaneously the decisions of how often to visit a site and which subset of sites to visit in positive quantities.
- model the number of visits to a given site as a function of its own price and prices of other visited sites. The number of visits depends on the prices of unvisited sites, but in a very structured fashion, i.e., through the corner selection function $\delta_\omega(\mathbf{p}, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$ and not through the conditional demand function $\mathbf{x}_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \boldsymbol{\epsilon}^{\text{int}})$.
- model the number of visits to a given site as a function of its own quality and the qualities of other visited sites. However, if weak complementarity is assumed, the qualities of unvisited sites will enter only through δ_ω and not through the conditional demand $\tilde{\mathbf{x}}_\omega(\mathbf{p}_\omega, y, \mathbf{q}_\omega, \boldsymbol{\epsilon}^{\text{int}})$.
- specifically incorporate the integer (count) nature of the data in the model.

Three Empirical Models

The three empirical models we investigate are (*a*) the repeated nested logit (RNL) model introduced by Morey, Rowe, and Watson; (*b*) a system of demand equations such as those estimated by Burt and Brewer; and (*c*) a KT model (Phaneuf, Kling, and Herriges). We chose these three models because they each have a claim to being utility theoretic and they represent well the range of alternative empirical models analysts have implemented.

Repeated Nested Logit Model

The RNL model is based on the random utility framework attributable to McFadden. In this model, recreationists are assumed to choose the alternative that yields the highest utility on any given choice occasion. By specifying a particular utility function and an error distribution, the analyst forms probability statements concerning the likelihood of a recreationist visiting a particular site on any given choice occasion, which provides the basis for maximum-likelihood estimation.

The model proceeds by specifying a conditional indirect utility function associated with each alternative: $V_j = V_j(y_c - p_j, q_j) + \epsilon_j$ $(j = 1, \ldots, M)$, where y_c is the per-choice-occasion income. The utility associated with not taking a recreational trip on any given choice occasion is similarly specified as $V_0 = V_0(y_c) + \epsilon_0$. On each choice occasion, the recreationist chooses whether to visit a recreation site and, if so, which site to visit, depending on which of these alternatives yields the highest utility. The alternatives can be nested in any number of ways.

Strictly speaking, the RNL model is consistent with utility theory in that it follows McFadden's random utility maximization (RUM) hypothesis. In addition, the structure of the model is consistent with the theoretical discussion in that only the price and quality of the chosen good enters the indirect utility function for each choice occasion. The integer nature of the data is accurately reflected in that the model provides count data predictions of how many trips are made in a season and the number of visits to each site. Two well-known criticisms of the model, however, are that for estimation purposes a fixed number of choice occasions are assumed during the season and that the individual's decisions are assumed to be independent across choice occasions. Recent developments in the use of the random-parameter logit (RPL) model (Train, McFadden and Train) provide a potential answer to the independence criticism.

The RPL model generalizes a standard multinomial logit model by allowing coefficients to vary randomly rather than fixing them. The model begins like the standard MNL model, specifying that the utility received by individual n during choice occasion t from selecting alternative j is given by $V_{njt} = V(x_{njt}, \epsilon_{njt}; \boldsymbol{\beta}_{nt}) = \boldsymbol{\beta}_{nt}x_{njt} + \epsilon_{njt}$, where ϵ_{njt} is an i.i.d. extreme value disturbance term. The difference is that the parameter vector $\boldsymbol{\beta}_{nt}$ is not assumed to be constant for all n and t. Instead, the $\boldsymbol{\beta}_{nt}$'s are treated as random parameters drawn from a distribution, known to the consumer but unobserved by the analyst. Train suggests that correlation across choice occasions can be introduced by assuming that $\boldsymbol{\beta}_{nt} = \boldsymbol{\beta}_n \; \forall \; t$. Given this assumption, and conditional on the parameter vector $\boldsymbol{\beta}_n$ for individual n, the probability of observing individual n's sequence of T choices is the product of the standard multinomial probabilities for each choice occasion:

$$(3) \qquad Y_n(\boldsymbol{\beta}_n) = \prod_{t=1}^{T} \frac{\exp(\boldsymbol{\beta}_n x_{j(n,t)})}{\sum_j \exp(\boldsymbol{\beta}_n x_j)}$$

where $j(n,t)$ indexes the alternative chosen by individual n on choice occasion t. The unconditional probability (P_n) is obtained by integrating this product over all values of β; i.e., $P_n(\theta) = \int Y_n(\beta)f(\beta \mid \theta)\, d\beta$, where $f(\beta \mid \theta)$ is the assumed probability density function for β, parameterized by θ. Finally, the log-likelihood function for all individuals in the sample is $L(\theta) = \Sigma_n \ln P_n(\theta)$. Simulation methods are then used to conduct maximum-likelihood estimation of the θ values (see Train).

There are several advantages of the RPL model over its MNL counterpart. First, it does not exhibit the restrictive "independence of irrelevant alternatives" property that is characteristic of MNL. Indeed, like nested logit, alternatives can be "grouped" together by introducing a dummy variable to the set of explanatory variables (i.e., the x_{njt}'s) identifying a specific collection (or nest) of alternatives. Second, the RPL model allows for a wide variation in preferences, yielding an estimated distribution of the marginal impact of each explanatory variable. Finally, in a panel data setting, the RPL model allows for explicit correlation among choice occasions for a given individual. For these reasons, we propose combining the RNL and RPL models, yielding what we will refer to below as the random parameters, repeated nested logit (RPRNI) model.

Systems of Demands

A second approach is to specify a system of demand equations (or share equations) and employ an estimator that explicitly accounts for the abundance of zeros. Recent studies have investigated estimators that account for censoring of the sample and/or the count nature of the data. A system of demand equations can be specified in general form as

$$(4) \qquad \begin{aligned} x_1 &= x_1(\mathbf{p}, \mathbf{q}, y) + \epsilon_1 \\ &\vdots \qquad\quad \vdots \qquad\quad \vdots \\ x_M &= x_M(\mathbf{p}, \mathbf{q}, y) + \epsilon_M. \end{aligned}$$

Note that in this traditional specification of the system, each demand function is assumed to depend upon all prices and qualities, regardless of whether corner solutions are present.

An adaptation of this basic systems model that incorporates the changes in demand function structure associated with corner solutions can be easily specified using a set of dummy variables. To illustrate, suppose there are only two demand functions in our system and that they are specified to be linear:

$$(5) \qquad \begin{aligned} x_1 &= \alpha_1 + \beta_{11}p_1 + \beta_{12}d_2p_2 + \gamma_{11}q_1 \\ &\quad + \gamma_{12}d_2q_2 + \epsilon_1 \\ x_2 &= \alpha_2 + \beta_{21}d_1p_1 + \beta_{22}p_2 + \gamma_{21}d_1q_1 \\ &\quad + \gamma_{22}q_2 + \epsilon_2 \end{aligned}$$

where $_{di}$ = 1 if $x_i > 0$; $d_i = 0$ otherwise. Note that the specification in equation (5) imposes weak complementarity as well.

Kuhn-Tucker Model

The KT model was originally proposed by Wales and Woodland as an estimation method for data with many binding nonnegativity constraints. It begins with the maximization of a random utility function, which generates KT conditions that are also random. From these conditions, probabilistic statements can be made regarding the observed outcome in the data. Formally, the consumer solves

$$(6)\quad \max_{x,z} U(\mathbf{x}, z, \mathbf{q}, \gamma, \epsilon)$$
$$\text{s.t.}$$
$$\mathbf{p}'\mathbf{x} + z = y, \qquad z \geq 0, \qquad x_j \geq 0,$$
$$j = 1, \ldots, M$$

where $U(\cdot)$ is assumed to be a quasi-concave, increasing, and continuously differentiable function of $(\mathbf{x}, z)$, $\epsilon = (\epsilon_1, \ldots \epsilon_M)'$ is a vector of random disturbances capturing the variation in preferences in the population, and **x**, **p**, **q**, and γ are as defined above. Given assumptions on the structure of the utility function, the standard KT conditions for utility maximization can be algebraically manipulated and conveniently expressed in the form

$$(7)\quad \epsilon_j \leq g_j(\mathbf{x}, y, \mathbf{p}, \mathbf{q}, \gamma); \qquad x_j \geq 0;$$
$$x_j[\epsilon_j - g_j(\mathbf{x}, y, \mathbf{p}, \mathbf{q}, \gamma)] =$$
$$0, j = 1, \ldots, M.$$

This form of the first-order conditions, along with a specification of the density function $f(\epsilon)$ for ϵ, provides the necessary information to construct the probability of the outcome in the data. Each individual in the data is assigned a probability based on that individual's observed visits, and maximum likelihood is used to recover estimates of the utility function parameters. Additional details on deriving and specifying the KT model can be found in Phaneuf, Kling, and Herriges.

The KT model simultaneously predicts the number of trips made and which subset of sites is visited. The integrated behavioral and econometric model automatically enforces the theoretical constraints of the corners model; i.e., only prices of consumed goods enter the demand equations. However, the KT model is still somewhat of a challenge to implement. Estimation becomes difficult with more than a small number of goods, and so far, only relatively restrictive utility functions have been used in applications of the model. In addition, the integer nature of recreation data has not yet been addressed in the KT model.

Application

The focus of the empirical analysis is angling in the Wisconsin Great Lakes. The usage data come from two mail surveys of angling behavior conducted in 1990 at the University of Wisconsin-Madison.[1] A total of 487 completed surveys were available for analysis, including 240 from individuals who had visited one or more of the twenty-two destinations identified for the Wisconsin Great Lakes region and 247 from individuals who fished only inland waterways (i.e., nonusers from the perspective of the Great Lakes region). We have combined the destinations of Great Lakes anglers into four aggregate sites: Lake Superior, South Lake Michigan, North Lake Michigan, and Green Bay. The price of a trip to each of the fishing sites consists of both the direct cost of getting to the site (round-trip travel cost) and the opportunity cost of the travel time (one third of the wage rate).

To characterize the demand, household income is included, along with a dummy variable to indicate ownership of a Great-Lakes-suitable boat. The two quality variables are toxin levels in lake trout flesh and a fishing catch rate index. For each site, the catch rate index is formed as the sum of the catch rates for four aggressively managed salmonoid species.[2]

Model Specifications

The estimation of the models identified above first requires the specification of the functional forms for both the stochastic and nonstochastic portions of each model. For the basic RNL model, the conditional indirect utilities per choice occasion for the four sites are specified as $V_j = \beta_1(y_c - p_j) + \beta_2 Catch_j + \beta_3 Toxin_j + \epsilon_j$ $(j = 1, \ldots, 4)$. The indirect utility associated with not making a recreation trip is

[1] Details of the sampling procedures and survey design are provided in Lyke.

[2] Details regarding the formation of site prices, catch rates, and toxin variables are provided in Phaneuf.

assumed to be $V_0 = \beta_0 + \beta_1 y_c + \beta_4 Boat + e_0$, where y_c is per-choice-occasion income, and it is assumed that there are fifty choice occasions during the season. The stochastic elements are drawn from a generalized extreme value distribution that yields a three-level nest. The first level is the binary choice of whether or not to make a trip, while the lower levels group the recreation sites, with North Lake Michigan and Green Bay in one nest and South Lake Michigan and Lake Superior in another.

The utilities associated with visiting each site and not making a trip are the same in the RPRNL model with two exceptions. First, the stochastic terms are now assumed to be distributed extreme value rather than generalized extreme value. Second, coefficients are assumed to be random. Here, we assume that $\beta_k \sim N(b_k,\sigma_k^2)$ ($k = 0, 2, 3, 4$). The marginal utility of income term (β_1) is assumed to be constant across households.

The systems model takes a linear form, with the demand for site j given by $x_j = \alpha_j + \beta_{jj}p_j + \Sigma_{k\neq j}\ \beta_{jk}d_k p_k + \gamma_{jj}q_j + \Sigma_{k\neq j}\ \gamma_{jk}d_k q_k + \epsilon_j (j = 1, \ldots, 4)$. The errors are assumed to be drawn from a truncated GEV distribution with the same nesting structure as in the RPL model.

Finally, the empirical KT model starts with the specification of the utility function as a version of the linear expenditure systems, with $U(\mathbf{x}, z, \mathbf{q}, \gamma, \epsilon) = \Sigma_{j=1}^{4}\ \Psi_j(q_j,\epsilon_j)\ln(x_j + \theta) + \ln(z)$, where $\Psi_j(q_j,\epsilon_j) = \exp(\delta_0 + \delta_1 Catch_j + \delta_2 Toxins_j + \delta_3 Boat + \epsilon_j)$. Again, the errors are assumed to be drawn from a GEV distribution with the same nesting structure as in the RNL model.

Results

Although space constraints prohibit the presentation of all the resulting parameters, table 1 provides estimates of select parameters so as to provide a flavor of the performance of each model. Table 1 indicates the effects of the site attributes of price, catch rate, and toxin levels, as well as the consumer attribute of boat ownership. All four specifications yield parameter estimates that are in line with expectations. Higher prices generally lead to a lower level of utility associated with a given site or reduction in the direct demand for that site. Likewise, higher catch rates enhance the desirability of a site, and demand for recreation is higher for those who currently own a boat. However, the four models are less uni-

Table 1. Key Parameter Estimates

	Parameter Estimates[a]							
		RPRNL			System			
Variable	RNL	Mean	Standard Deviation	KT	Site 1	Site 2	Site 3	Site 4
Price[b]	−0.003[c]	−0.01[c]	na	na	−0.04[c]	0.00	−0.02	−0.05[c]
	(0.000)	(0.00)			(0.01)	(0.01)	(0.02)	(0.01)
Catch rate	1.99[c]	14.40[c]	24.07[c]	7.11[c]	29.63[c]	50.80[c]	25.81[c]	99.34[c]
	(0.07)	(0.87)	(0.74)	(0.81)	(4.98)	(8.39)	(23.31)	(10.00)
Toxin	−0.05[c]	−0.93[c]	0.97[c]	−0.07[c]	1.36	−0.16	0.14	−4.19[c]
	(0.00)	(0.04)	(0.03)	(0.02)	(1.28)	(0.23)	(0.31)	(1.58)
Boat	−1.55[c]	−1.42[c]	0.15	−1.36[c]	1.44	6.35[c]	6.19[b]	10.21[c]
	(0.01)	(0.14)	(0.18)	(0.21)	(1.48)	(1.52)	(1.89)	(1.55)

[a] t-statistics are given in parentheses.
[b] For the systems model, price refers to the own price. For the RNL and RPRNL models, the price coefficients correspond to the negative of β_1.
[c] Statistically different from zero at a 99% confidence level.

Table 2. Welfare Estimates

	RNL	RPRNL	KT	System
Scenario A: 20% reduction in toxins	−29.16	−8.78	−116.45	10.99
Scenario B: loss of South Lake Michigan	162.67	98.34	849.09	309.21

Note: In dollars per angler per season.

form in terms of measuring the impact of toxins on recreation demand. The RNL, RPRNL, and KT models indicate that toxins significantly reduce the utility or quality of a site. This is less clear, however, in the systems model. For sites 2 and 4, higher toxin levels reduce recreation demand (significantly for site 4). However, toxins have a positive, although insignificant, effect on the other sites.[3]

While the parameters of the four models are not directly comparable, they can each be used to evaluate the welfare effects of changes in water quality in the Great Lakes fishery. For each of the estimated models, the welfare effects of the following two scenarios are examined:

- *Scenario A: 20% reduction in toxins at all sites.* Improved industrial and municipal waste management leads to a general improvement in water quality at all sites, resulting in a decrease in toxin levels.
- *Scenario B: loss of South Lake Michigan.* Due to an environmental disaster, South Lake Michigan is no longer suitable for recreation fishing and is eliminated from the choice set.

Table 2 provides estimates of the compensating variation associated with each scenario for each of the models.[4] Not surprisingly, the four models yield a wide range of welfare estimates. For toxins, the RPRNL, KT, and RNL models yield the expected welfare gain (i.e., a negative compensating variation) associated with reduced toxin levels. This is consistent with the parameter estimates provided in table 1. All four models suggest that the loss of the South Lake Michigan site will substantially reduce consumer welfare, although the estimated compensating variations range from a low of $98.34 for the RPRNL model to a high of $849.09 for the KT model. One explanation for the larger welfare estimates for the KT model is that they include non-use (but not existence) value, whereas the other models reflect only direct-use value.

Conclusions

As mentioned above, each of these four approaches can claim in some sense to be utility theoretic. At this point, it appears that the KT model comes closest to matching the ideal theoretical model developed in the first section. The relative difficulties associated with its use, however, may preclude its application in every situation. In contrast, the RNL model is easy to implement but relies on restrictive assumptions. Appealing, and worthy of additional research beyond this presentation, is the use of RPL model as an analog to the RNL model. Likewise, systems of demand equations that account for the structural changes associated with corners are worthy of further research.

[3] Another aspect of the RPRNL parameter estimates worth noting is that estimated variability in random parameters (i.e. σ_i's) is statistically significant, implying significant correlation in site selection across choice occasions, a feature not captured by the basic RNL model.

[4] Details regarding the computation of the welfare measures are available from the authors.

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