Journal of Environmental Economics and Management **42,** 53–64 (2001)
doi:10.1006/jeem.1149.2000, available online at http://www.idealibrary.com on IDEAL®

Markets for Water Rights under Environmental Constraints[1]

Marian L. Weber

Department of Rural Economy, University of Alberta, Edmonton, Alberta T6G 2H1, Canada
E-mail: marian.weber@ualberta.ca

Received April 25, 1999; revised October 20, 1999; published online January 19, 2001

The cumulative impact of pollutants on water quality at a given location along a river is a function of prior loadings, as well as pollution and surface flows at that particular site. The question of whether it is possible to maintain water quality under alternative mechanisms for allocating surface water and pollution rights is addressed. We model the optimal allocation of surface water and pollution rights along a river with water quality constraints. Due to cumulative effects, the shadow values of water and pollution rights are contingent on location. It is shown that a Nash equilibrium in a market for tradable surface water rights in conjunction with a market for pollution damages implements the optimal solution. The market supports location-specific prices for both consumptive water rights and pollution damages.

1. INTRODUCTION

The merits of alternative mechanisms for allocating surface water rights are important given increasing demands on existing fresh water sources. Many water sources are fully allocated and there are pressures to reallocate water to suit changing economic conditions. The allocation problem is compounded by environmental concerns. Minimum instream flows are required to maintain wildlife and recreation values, while pollution from agricultural, industrial, and municipal sources erodes the quality of flows. The use of tradable permits for allocating surface water rights is attractive. Politicians are not seen to arbitrarily assign winners and losers in the allocation process, while at the same time market incentives encourage reallocation of water to higher benefit uses [3]. Efficiency gains from tradable water rights appear to be significant. Wong and Eheart [18] find that market systems recover approximately 95% of the economic value of water. Tradable permit systems are also attractive mechanisms for regulating water quality and are slowly being implemented in the U.S. [2]. The EPA's policy of promoting tradable permits for effluent discharges under the Clean Water Act has led to the development of experimental trading programs in several watersheds.

Two key issues arise in assessing the efficiency of tradable permits for allocating surface water rights in river systems as opposed to lake-like systems. First, flow constraints between users can become binding. This occurs when a reallocation of

[1] I thank Robin Lindsey for his valuable feedback. I also thank Vic Adamowicz, Brian Copeland, Henry van Egteren, Grant Hauer, Ted Horbulyk, and two anonymous reviewers for their comments and suggestions. An earlier version of this paper was presented to the Canadian Resource and Environmental Economics Study Group meeting in October 1999.

0095-0696/01 $35.00

consumptive rights from downstream to upstream users reduces flows to intermediate users to the extent that they cannot divert enough water to fulfill their consumptive rights. These so-called third party effects arise when return flows are used to expand the total volume of water available for diversion in the river system.[2] Second, the capacity of a river to absorb pollutants depends on the magnitude of surface flows. Therefore a redistribution of surface rights will alter water quality along the river.

Efficiency aspects of markets allocating surface rights along a river have been addressed previously (see [1, 3, 4, 9, 10]). The results of this literature show that permits which specify rights for consumptive use implement the optimal solution as long as flow constraints between users are not binding. This condition is automatically satisfied if return flows are not allocated. However, when return flows are allocated, the shadow value of a consumptive water right becomes location-specific due to the dependence of downstream consumptive rights on upstream return flows. Therefore a market in which water rights trade at a single price is generally inefficient [10]. Similarly, when there are instream flow demands, the actions of individual right holders are not optimal because of the costs that accrue to multiple downstream users as a result of increased consumption at a particular location [1].

The efficiency of markets for allocating pollutants depends on whether permits are defined in terms of emissions or damages. Permit systems defined in terms of emissions generally are not efficient since they do not capture the spatial characteristics of damages from firms [14, 15]. Modelers frequently avoid the spatial problem by assuming that marginal social damages from pollution are identical for all firms. However, this assumption is inappropriate when applied to the problem of river pollution. For many categories of pollutants cumulative effects result in higher marginal damages and restrict input choices for downstream users. Therefore the shadow value of a pollution right depends on the location of the polluter along the river.

These results suggest that a tradable permit system must be capable of supporting a vector of location-specific prices for both surface water and pollution rights. In addition, since the level of water quality depends on both surface flows and discharges, a market for consumptive water rights must also internalize impacts on water quality. In this paper we characterize the optimal allocation of pollution and consumptive water rights along the river. Users are assumed to derive benefits from instream flows and quality as well as consumptive rights and emissions. We show that a market for consumptive water rights coupled with a Montgomery-style market allocating an aggregate level of pollution damages in the watershed generates the optimal outcome. We find that the marginal benefit of purchasing a consumptive water right depends on the location of the seller. In particular, if a permit is purchased from an upstream seller, both water flows and water quality increase at the buyer's point of diversion. On the other hand, purchasing a permit from a downstream seller does not generate equivalent flow or quality benefits. Analogous results hold for pollution permits. This effectively divides the market into an upstream and downstream segment at each location since instream flow demands are incorporated into the willingness to pay for upstream permits. We use

[2] Return flows are defined as the difference between the amount of water diverted and the amount consumed. Since return flows can be re-diverted downstream, the total amount of water available for diversion in the watershed exceeds the flow at the source.

this property to show that a Nash equilibrium supports optimal location-specific prices.

The remainder of the paper proceeds as follows. In Section 2 we solve for the efficient allocation of water and pollutants along the river. In Section 3 we analyze the outcome of a tradable permit system which allocates both consumptive water rights and pollution damages. In the main proposition of the paper we show that a Nash equilibrium supports efficient location-specific prices. The result requires the bidding process to be completely transparent; that is, all potential buyers and sellers must be aware of each others' locations and bids. This suggests that transaction costs will be important. We conclude by discussing the feasibility of the solution and suggest ideas for future research.

2. OPTIMAL ALLOCATION OF WATER AND POLLUTANTS

Let the set of water users located along a river be ordered as $i = 1, \dots, n$, by increasing distance from the source. Water flows at the source are denoted by ν_0. Assume that there are no branches feeding into the system. Then the amount of water available for diversion at the ith location, $\nu(i)$, depends only on the consumption of water by upstream users.[3] The total amount of water consumed by user i is equal to

$$c(i) = (1 - R^i) s(i),$$

where $s(i)$ and $c(i)$ are the amounts of water diverted and consumed, respectively, and R^i is an exogenously determined return flow parameter.[4] Therefore the amount of water available for diversion at any point along the river evolves according to the first-order difference equation

$$\nu(i+1) - \nu(i) = -(1 - R^i) s(i).$$

As in [16] we assume that water quality at any reach downstream from a specified point source of pollution is determined by a first-order difference equation. Let $q(i)$ denote the level of water quality at user i's intake and let $e(i)$ denote the amount of effluent that user i chooses to discharge.[5] Then the difference in water quality between users i and $i+1$ can be written as the site-specific function

$$q(i+1) - q(i) = f^i(c(i), e(i), \nu(i), q(i)).$$

Assumption 1 (A1). $f^i(c(i), e(i), \nu(i), q(i))$ is decreasing and strictly concave in c and e, and increasing and strictly concave in ν and q for all users.

[3] We assume that water flows are not stochastic in order to highlight the problems which arise from separating the regulation of water consumption from water quality. A regime which cannot support the optimum in a deterministic setting will not support the optimum in a stochastic setting.

[4] Burness and Quirk [4] provide ranges for return flow parameters associated with particular water uses. These R values range from 5–10% for evaporative cooling to 30–60% for agricultural use and 80–90% for domestic and municipal use.

[5] We assume that the level of effluent produced by user i can be captured by a single dimension variable e which is a monotonically increasing function of the level of effluent-generating inputs.

We assume that water quality is decreasing in water consumption and discharges so that $f_c^i < 0$ and $f_e^i < 0$. We also assume that an increase in quality or flows at site i increases the assimilative capacity of the river at site i. Therefore, $f_q^i > 0$ and $f_\nu^i > 0$. Note that this specification allows water quality to improve between users. That is, it is possible for $f^i(\cdot) > 0$, even when $c(i)$ and $e(i)$ are positive. The level of overall quality at any intake for user i is then

$$q(i) = \sum_{j=1}^{i-1} f^j(c(j), e(j), \nu(j), q(j)) + q_0.$$

The allocation problem is constrained by two factors. First, endpoint constraints stipulate the minimum quantity $\bar{\nu}$ and quality $\bar{q}$ of water which must be left after the nth user. These constraints are determined by compact agreements negotiated between jurisdictions. Second, instream flow need (IFN) constraints specify minimum levels of quality and flows, $\tilde{q}$ and $\tilde{\nu}$, which must be maintained along the river. These constraints are assumed to be optimally set by the regulator. The IFN constraints on water flows and quality can be stated as

$$\nu(i) - c(i)/(1 - R^i) \geq \tilde{\upsilon},$$

$$q(i) + f^i(c(i), e(i), \nu(i), q(i)) \geq \tilde{q}.$$

Third party effects result when transfers of water and pollution rights from downstream to upstream users cause these constraints to become binding for intermediate users.[6]

Each user's benefits depend positively on the amount of water consumed, the effluent discharged, and the quality and quantity of water at the point of diversion. Low water flows and quality may increase diversion and operating costs. The benefit function for user i is written

$$B^i = B^i(c(i), e(i), \nu(i), q(i)).$$

Assumption 2 (A2). The benefit function is increasing and strictly concave in c, e, ν, and q.[7]

Assumption 3 (A3). Input consumption is strictly positive for all users.

According to this formulation, all users must derive positive benefits from water consumption. The results of the model do not extend to the case where some users value water solely for non-consumptive uses. We assume that non-consumptive values are captured by the regulator in setting instream flow constraints.

The regulator maximizes the total benefits from allocating water and pollution rights along the river by solving the control problem

$$\max_{s(i), e(i)} \sum_{i=1}^{n} B^i(c(i), e(i), \nu(i), q(i)), \tag{P1}$$

[6] In models without explicit IFN requirements on water flows, $\tilde{\nu} = 0$ (cf. [10]). The IFN constraint in this case simply states that there cannot be negative flows.

[7] It is assumed that pollution-producing inputs and water inputs are substitutes. For industries such as agriculture, these inputs are sometimes assumed to be consumed at fixed ratios. Leontief benefit functions result in allocations which are corner solutions. In order to focus on the marginal impacts of water reallocation, we ignore the case of Leontief benefit functions.

subject to

$$\nu(i+1) - \nu(i) = -(1-R^i)s(i) \qquad (\mu_1(i) \geq 0); \tag{P1.1}$$

$$q(i+1) - q(i) = f^i(c(i), e(i), \nu(i), q(i)) \qquad (\mu_2(i) \geq 0); \tag{P1.2}$$

$$\nu(i) - s(i) - \tilde{\nu} \geq 0 \qquad (\lambda_1(i) \geq 0); \tag{P1.3}$$

$$q(i) + f^i(c(i), e(i), \nu(i), q(i)) - \tilde{q} \geq 0 \qquad (\lambda_2(i) \geq 0); \tag{P1.4}$$

$$\nu(1) = \nu_0, \qquad \nu(n+1) = \bar{\nu}, \qquad q(1) = q_0, \qquad q(n+1) = \bar{q}; \tag{P1.5}$$

$$s(i) \geq 0, \qquad e(i) \geq 0.^8 \tag{P1.6}$$

This is a general constrained control problem with two control variables, s and e, and two state variables, ν and q. The constraints (P1.1) and (P1.2) are state equations which describe the evolution of flows and quality in the system, (P1.3) and (P1.4) are the IFN constraints on flows and quality, respectively, and (P1.5) represents the initial and endpoint conditions. Assumptions (A1)–(A3) guarantee an interior solution to the control problem (see [13]). The necessary conditions for (P1) are set out in Eqs. (1)–(8) below:

$$B_c^i - \mu_1(i) + \mu_2(i)f_{c(i)}^i - \lambda_1(i)/(1-R^i) + \lambda_2(i)f_{c(i)}^i \leq 0; \tag{1}$$

$$B_e^i + \mu_2(i)f_{e(i)}^i + \lambda_2(i)f_{e(i)}^i \leq 0; \tag{2}$$

$$\mu_1(i) - \mu_1(i-1) = \Delta\mu_1(i) = -\left[B_\nu^i + \lambda_1(i) + (\mu_2(i) + \lambda_2(i))f_{\nu(i)}^i\right]; \tag{3}$$

$$\mu_2(i) - \mu_2(i-1) = \Delta\mu_2(i) = -\left[B_q^i + \lambda_2(i) + (\mu_2(i) + \lambda_2(i))f_{q(i)}^i\right]; \tag{4}$$

$$\nu(i) - s(i) - \tilde{\nu} \geq 0, \qquad \lambda_1(i)[\nu(i) - s(i) - \tilde{\nu}] = 0; \tag{5}$$

$$q(i+1) - \tilde{q} \geq 0, \qquad \lambda_2(i)[q(i+1) - \tilde{q}] = 0; \tag{6}$$

$$\nu(n) - (1-R^i)s(n) - \bar{\nu} \geq 0, \qquad q(n+1) - \bar{q} \geq 0; \tag{7}$$

$$\frac{\partial \mathscr{L}}{\partial s(i)} s(i) = 0, \qquad \frac{\partial \mathscr{L}}{\partial e(i)} e(i) = 0. \tag{8}$$

Making the appropriate substitutions we can describe the imputed prices of water consumption and discharges at each site:

$$\mu_1(i) = B_c^i - (B_e^i/f_e^i)f_c^i - \lambda_1(i)/(1-R^i); \tag{9}$$

$$\mu_2(i) = -(B_e^i/f_e^i) - \lambda_2(i); \tag{10}$$

$$\Delta\mu_1(i) = -\left[B_\nu^i + \lambda_1(i) - (B_e^i/f_e^i)f_\nu^i\right]; \tag{11}$$

$$\Delta\mu_2(i) = -\left[B_q^i + \lambda_2(i) - (B_e^i/f_e^i)f_q^i\right]. \tag{12}$$

[8] Note that as long as the benefit functions are monotonically increasing in $e(i)$ and $c(i)$, the compact constraints $\bar{q}$ and $\bar{\nu}$ will be binding.

The left hand side of Eq. (9) shows the cost of reducing flows to downstream users by one unit. At the optimum, this is set equal to the *net* marginal benefit of allocating an extra unit of water to user i. The first term on the RHS is the marginal benefit of consuming an extra unit of water while the second term represents the increased cost to user i of meeting the quality constraint due to the resulting decrease in flows. Finally the third term on the RHS represents the value of the IFN constraint on water flows to user i. This constraint may or may not be binding. Similarly Eq. (10) shows that at the optimum the shadow cost of reducing quality to downstream users by one unit is just equal to the *net* marginal benefit of emissions. If the IFN constraint on quality is not binding this is just equal to the marginal benefit of the discharge weighted by the marginal damage caused from emitting an extra unit of effluent.

Equations (11) and (12) show the evolution of the costate variables μ_1 and μ_2 and determine the optimal price paths for water and pollution rights. Given (A1) and (A2), $\Delta\mu_1(i) < 0$ and $\Delta\mu_2(i) < 0$, reflecting the fact that upstream input use generates negative externalities for all downstream users through the state variables $\nu(i)$ and $q(i)$. Since fewer users are affected by these externalities as one travels downstream, the social cost of input use declines from upstream to downstream *ceteris paribus*. If $\lambda_1(i) \neq 0$ (resp. $\lambda_2(i) \neq 0$), then from (3) (resp. (4)) we see that there is a discrete drop in the shadow cost of allocating water (resp. damages) to downstream users. This is because a shift in input use from upstream to downstream of i increases the volume and quality of flows at site i, thus reducing i's IFN constraints. Since $\mu_1(i)$ and $\mu_2(i)$ decrease with i, any market which supports the optimal solution will have permit values which decline from upstream to downstream.

3. MARKET EQUILIBRIUM

We now address the question of whether a decentralized market for water and pollution rights is efficient when there are water quality constraints. The market for tradable water rights is described as follows. The regulator allocates an aggregate level of water permits $\overline{W} = \sum_{i=1}^{n} w_i^0$, where w_i^0 is the initial allocation to each user i and $\overline{W} = \nu_0 - \bar{\nu}$ is the total amount of water available for consumption. User i is in compliance when $w(i) \geq c(i)$. This constraint holds with equality in equilibrium as long as permit prices are positive.[9] The regulator also allocates permits for pollution damages $\overline{D} = \sum_{i=1}^{n} d_i^0$ where $\overline{D} = q_0 - \bar{q}$. If $f^i(\cdot) > 0$, then user i generates a credit which also is tradable. Let $d(i)$ be the total number of pollution rights held by user i. Compliance requires that $d(i) \geq -f^i(c(i), e(i), \nu(i), q(i))$.[10] We assume that the market is perfectly competitive.

[9] The result is proved formally in [15, Theorem 3.2]. However, Montgomery's proof does not automatically hold for consumptive water permits. Since the right to consume water is not equivalent to the right to divert water, there may be cases (for example, when return flow parameters across users are "high enough") where no user can divert enough water to exhaust consumptive rights without violating the IFN constraint $\tilde{\nu}$. Note $R^i = 0$ implies that the required diversion is just equal to the consumptive right. If $R^i = 0$, and the marginal benefit of water use is positive for some user i, then that user will pay positive prices for permits from any location and no user will hold excess permits in equilibrium.

[10] When $f^i(c(i), e(i), \nu(i), q(i)) < 0$, damages occur and $d(i) > 0$. On the other hand, if quality improves downstream, $d(i) < 0$ and a credit is generated.

Let w_i^k and d_i^k be the numbers of permits purchased by user i from user k. Negative purchases are interpreted as sales from user i to user k. The final holdings for each user are defined by $w(i) \equiv w_i^0 + \sum_{k=1}^n w_i^k$ and $d(i) \equiv d_i^0 + \sum_{k=1}^n d_i^k$. The number of permits purchased from k depends on p_{wi}^k and p_{di}^k, the prices paid by i for permits from location k. If $w_i^k < 0$ (resp. $d_i^k < 0$), then p_{wi}^k (resp. p_{di}^k) represents the reservation price at which user i is willing to sell water (resp. damage) rights to k. No one holds excess permits in equilibrium.

The evolution of flows and quality along the stream can be written as $\nu(i) = \nu_0 - \sum_{k=1}^{i-1} w(k)$, and $q(i) = q_0 - \sum_{k=1}^{i-1} d(k)$. Since $\partial w(k)/\partial w_i^k = \partial d(k)/\partial d_i^k = -1$, users control the volume and quality of water at their intake through the purchase (sale) of permits from (to) upstream users. However, $\nu(i)$ and $q(i)$ do not depend on downstream input use; therefore an asymmetry is introduced with respect to the willingness to pay for upstream versus downstream permits.

The market described above is a pure-exchange Walrasian market. Each user bids for permits by offering a price at which he or she is willing to pay for a permit from a given location. At the same time each user identifies a reservation price at which he or she will sell permits. The market is in equilibrium when there is no incentive for any pair of users to trade. The objective of each user is to choose a portfolio of permits which maximizes benefits subject to the compliance and IFN constraints. Therefore each user chooses vectors $w = (w_i^1, \ldots, w_i^{i-1}, w_i^{i+1}, \ldots, w_i^n)$ and $d = (d_i^1, \ldots, d_i^{i-1}, d_i^{i+1}, \ldots, d_i^n)$ to solve the programming problem

$$\max_{c,e,w,d} B^i(c(i), e(i), \nu(i), q(i)) - \sum_{k \neq i} p_{wi}^k w_i^k - \sum_{k \neq i} p_{di}^k d_i^k \tag{P2}$$

subject to the compliance constraints

$$\sum_{k \neq i} w_i^k + w_i^0 - c(i) = 0 \qquad (\gamma_1(i) \geq 0) \tag{P2.1}$$

$$\sum_{k \neq i} d_i^k + d_i^0 + f^i(c(i), e(i), \nu(i), q(i)) = 0 \qquad (\gamma_2(i) \geq 0) \tag{P2.2}$$

and the IFN constraints

$$\nu(i) - \frac{c(i)}{(1 - R^i)} - \tilde{\nu} \geq 0 \qquad (\gamma_3(i) \geq 0) \tag{P2.3}$$

$$q(i) + f^i(c(i), e(i), \nu(i), q(i)) - \tilde{q} \geq 0 \qquad (\gamma_4(i) \geq 0). \tag{P2.4}$$

Given (A1)–(A3), there is an interior solution to (P2) [13]. Now consider a bidding game between two users, i and j, for permits from a third user k. Assume that i wins the bid for k's water (resp. damage) permit when $p_{wi}^k > p_{wj}^k$ (resp. $p_{di}^k > p_{dj}^k$). We show that this bidding game results in the optimal allocation of water and pollution rights between i, j, and k. Since the result holds for any three users i, j, and k, it generalizes to all users in the system.

The payoffs to user i of the bidding game are set out below and are derived from the first-order conditions for (P2). Note that the payoffs of winning and losing are not symmetric and depend on the locations of both j (the competing bidder) and k (the seller). The payoff from winning a bid against j is just the marginal benefit of the permit minus the price that user i pays k. The payoffs of winning depend only on the location of k and are given in

$$B_c^i + B_v^i - \frac{B_e^i}{f_e^i}\left[f_c^i + f_v^i\right] - \frac{\gamma_3(i)R^i}{1-R^i} - p_{wi}^k, \qquad k < i; \tag{13}$$

$$B_q^i - \frac{B_e^i}{f_e^i}\left[1 + f_q^i\right] - p_{di}^k, \qquad k < i; \tag{14}$$

$$B_c^i - \frac{B_e^i}{f_e^i}f_c^i - \frac{\gamma_3(i)}{1-R^i} - p_{wi}^k, \qquad k > i; \tag{15}$$

$$-\frac{B_e^i}{f_e^i} - \gamma_4(i) - p_{di}^k, \qquad k > i. \tag{16}$$

Note that the payoff from purchasing an upstream right is greater than the payoff from purchasing a downstream right since upstream rights generate positive flow and quality benefits.

The payoff of losing a bid against j depends on the locations of both j and k. Any transfer from a downstream user k to an upstream user j will impose negative externalities on i due to a reduction in upstream flows and/or quality. These costs are given in

$$-B_v^i + \frac{B_e^i}{f_e^i}f_v^i - \gamma_3(i), \qquad j < i < k; \tag{17}$$

$$-B_q^i + \frac{B_e^i}{f_e^i}f_q^i - \gamma_4(i), \qquad j < i < k. \tag{18}$$

Alternatively, a transfer of rights from an upstream user k to a downstream user j increases flows and/or quality at site i and generates positive externalities

$$B_v^i - \frac{B_e^i}{f_e^i}f_v^i + \gamma_3(i), \qquad k < i < j; \tag{19}$$

$$B_q^i - \frac{B_e^i}{f_e^i}f_q^i + \gamma_4(i), \qquad k < i < j. \tag{20}$$

Finally, trades which redistribute rights between upstream or downstream users ($k, j < i$, or $k, j > i$) have no impact on i.

User i also can sell permits to k in which case the payoffs are given in

$$p_{wk}^i - B_c^i - B_v^i + \frac{B_e^i}{f_e^i}\left[f_c^i + f_v^i\right] + \frac{\gamma_3(i)R^i}{1-R^i}, \qquad k < i; \tag{21}$$

$$p_{dk}^i - B_q^i + \frac{B_e^i}{f_e^i}\left[1 + f_q^i\right], \qquad k < i; \tag{22}$$

$$p_{wk}^i - B_c^i + \frac{B_e^i}{f_e^i}f_c^i + \frac{\gamma_3(i)}{1-R^i}, \qquad k > i; \tag{23}$$

$$p_{dk}^i + \frac{B_e^i}{f_e^i} + \gamma_4(i), \qquad k > i. \tag{24}$$

Payoffs are positive when the bid price exceeds the net marginal benefit of the right. Again, the payoffs depend on the location of k, since selling permits upstream results in an overall reduction in water flows and quality at site i. Given the payoffs for buying and selling licenses in (13)–(24), we can show that a Nash equilibrium in the market for water and pollution rights results in the socially optimal outcome.

LEMMA 1. *A set of bid prices* $\sigma_i = [p_{wi}^1, \ldots, p_{wi}^k, \ldots, p_{wi}^n; p_{di}^1, \ldots, p_{di}^k, \ldots, p_{di}^n]$ *is a Nash equilibrium iff*

$$B_c^i - \frac{B_e^i}{f_e^i} f_c^i - \frac{\gamma_3(i)}{1 - R^i} = B_c^{i+1} + B_\nu^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[f_c^{i+1} + f_\nu^{i+1}\right] - \frac{\gamma_3(i+1) R^{i+1}}{1 - R^{i+1}} \tag{25}$$

$$-\frac{B_e^i}{f_e^i} - \gamma_4(i) = B_q^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[1 + f_q^{i+1}\right]. \tag{26}$$

Proof. Assume the contrary. Let

$$B_c^i - \frac{B_e^i}{f_e^i} f_c^i - \frac{\gamma_3(i)}{1 - R^i} < B_c^{i+1} + B_\nu^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[f_c^{i+1} + f_\nu^{i+1}\right] - \frac{\gamma_3(i+1) R^{i+1}}{1 - R^{i+1}}.$$

Then it is optimal for user $i+1$ to bid

$$p_{w(i+1)}^i \in \left[B_c^i - \frac{B_e^i}{f_e^i} f_c^i - \frac{\gamma_3(i)}{1 - R^i}, B_c^{i+1} + B_\nu^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[f_c^{i+1} + f_\nu^{i+1}\right] - \frac{\gamma_3(i+1) R^{i+1}}{1 - R^{i+1}}\right]$$

to buy a permit from i, and for i to sell. By the same argument, if

$$B_c^i - \frac{B_e^i}{f_e^i} f_c^i - \frac{\gamma_3(i)}{1 - R^i} > B_c^{i+1} + B_\nu^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[f_c^{i+1} + f_\nu^{i+1}\right] - \frac{\gamma_3(i+1) R^{i+1}}{1 - R^{i+1}},$$

then it is optimal for i to bid

$$p_{wi}^{i+1} \in \left[B_c^i - \frac{B_e^i}{f_e^i} f_c^i - \frac{\gamma_3(i)}{1 - R^i}, B_c^{i+1} + B_\nu^{i+1} - \frac{B_e^{i+1}}{f_e^{i+1}}\left[f_c^{i+1} + f_\nu^{i+1}\right] - \frac{\gamma_3(i+1) R^{i+1}}{1 - R^{i+1}}\right]$$

to buy a permit from $i+1$, and for $i+1$ to sell. Only when (25) holds with equality are there no gains from trade between adjacent users. Therefore (25) is a necessary condition for Nash equilibrium in the market for water rights. By analogy (26) is a necessary condition for Nash equilibrium in the market for damages.

For (25) and (26) to be sufficient conditions, we must show that there is no incentive for non-adjacent trades. Note that (25) implies $p_{wi}^{i-1} > p_{wi}^{i+1}$, and $p_{wi-1}^{i} > p_{wi+1}^{i}$. This means that downstream users are never willing to pay the opportunity cost for non-adjacent upstream permits at the margin. In addition, upstream users will never purchase permits from non-adjacent downstream users at equilibrium because there is no price which $i-1$ is willing to pay to $i+1$ that will not be matched by i. Therefore there is no incentive for non-adjacent trades in water permits if (25) holds. By analogy there is no incentive for non-adjacent trades in damage permits when (26) holds. Q.E.D.

PROPOSITION 1. *If $\overline{W} = v_0 - \bar{v}$ and $\overline{D} = q_0 - \bar{q}$, then the market for water and pollution damage permits implements the solution to* (P1).

Proof. The proof follows from Lemma 1 and the definition of $\overline{W}$ and $\overline{D}$. Q.E.D.

The proof follows from the fact that the reservation price at site i for selling water (resp. damage) permits downstream is equal to the marginal social cost of water use (resp. pollution) at site i. When users are required to hold damage licenses, they become responsible for changes in cumulative water quality and optimally substitute inputs according to the benefit functions and quality conditions at each site. Furthermore, all spillovers are internalized. When downstream users purchase upstream rights, they create positive benefits for all intermediate users. At the same time, intermediate users are willing to pay more for an upstream license than they are willing to accept to sell the same license downstream since by selling the license downstream they will still enjoy the benefits of increased flows and quality at their own site. Thus the positive benefits generated from shifting consumption downstream are fully captured through reduced prices to downstream buyers. Similarly, the external costs associated with transfers of downstream rights to upstream users are captured through the increasing prices for upstream licenses. The market not only efficiently allocates environmental inputs between users as in Montgomery [15], but also facilitates Coasean bargaining between users so that externalities are internalized [5]. A corollary of this result is that third party effects typically associated with water transfers are an equity rather than an efficiency issue.

4. CONCLUSIONS

We have derived conditions for maximizing the total benefits of water consumption and pollution in a river system when there are cumulative effects. The primary result of this paper is to show that a market serves both to allocate an aggregate level of environmental inputs, as well as internalize the spillover effects associated with input use as we move downstream. The market is efficient when users recognize that there are asymmetric benefits from purchasing upstream versus downstream rights. We use this quality to show that the market supports location-specific prices for environmental rights.

While the market is efficient, information requirements are high. Since pollution rights are defined in terms of damages rather than emissions, incremental damages must be measured at each site. If the regulator knows the water quality production function, then incremental damages can be calculated from site discharges. Even if there is a significant difference in the cost of measuring damages relative to discharges, this must be weighed against the efficiency gains of a market defined in terms of damages.

A second problem is that we do not explicitly consider how prices evolve or how information is transmitted in the market. Laissez-faire markets, in which the regulator takes a passive role in facilitating trades, tend to be inefficient in part because the transactions costs of finding trading partners are high [8, 11]. In our model a laissez-faire market does not provide enough information to all of the participants to generate efficient allocations. Each user must have, at any time, full knowledge of the bids and locations of all other users. One trading process which shows promise for addressing the information requirements of our model is the double auction. In the double auction initial allocations are grandfathered to each of the participants who then buy and sell from each other in a central organized market. All current bids, offers, and trades are public information, and each participant must outbid the others in order to retain its initial allocation. The sophistication of current electronic trading systems has the potential to greatly reduce the resulting transactions costs (see, for example, the Caltech Multiple Unit Double Auction described in [11]). This discussion illustrates the importance of the design of the trading system itself. Ledyard and Szakaly-Moore [11] provide a detailed discussion of issues involved in designing mechanisms for allocating pollution rights. Since there may be only a small number of users in a watershed, and bids are not anonymous, the potential for strategic interaction is an important concern. In addition, if users do not know each others' costs, then the sequence of bids may be important.

Still the greatest obstacle to implementing a trading system for water rights is the legal context in which water is currently allocated. States which allow trades in water do so with many restrictions attached (see [2, 6, 7, 12, 17]). In particular, protection from third-party injury is established under common law. At the same time, the definition of third-party damage is expanding to include the public interest. In our model, efficient trades impose costs on third parties if they have to adjust their portfolios. It is not clear how such a market would function in the current legal setting.

The main conclusion of this analysis is that water control agencies must consider the combined effects of water consumption and emissions when allocating surface water and pollution rights. In commenting on the feasibility of implementing the results from the model, we note that more research is necessary on the institutional setting in which rights will be traded. Finally, this framework may provide insight to similar problems where multiple pollutants interact systematically to produce cumulative effects.

REFERENCES

1. T. L. Anderson and R. N. Johnson, The problem of instream flows, *Econom. Inquiry* **24**, 535–554 (1986).
2. T. L. Anderson and P. Snyder, "Water Markets: Priming the Invisible Pump," Cato Institute, Washington, DC (1997).

3. H. S. Burness and J. P. Quirk, Appropriative water rights and the efficient allocation of resources, *Amer. Econom. Rev.* **69**, 25–37 (1979).
4. H. S. Burness and J. P. Quirk, Water law, water transfers, and economic efficiency: The Colorado River, *J. Law Econom.* **23**, 111–134 (1980).
5. R. H. Coase, The problem of social cost, *J. Law Econom.* **1**, 1–44 (1960).
6. B. G. Colby, Water reallocation and valuation: Voluntary and involuntary transfers in the western United States, *in* "Water Law: Trends, Policies and Practice" (K. M. Carr and J. D. Crammond, Eds.), American Bar Association, Section of Natural Resources, Energy and Environmental Law, Chicago, IL (1995).
7. G. A. Gould, Recent developments in the transfer of water rights, *in* "Water Law: Trends, Policies and Practice" (K. M. Carr and J. D. Crammond, Eds.), American Bar Association, Section of Natural Resources, Energy and Environmental Law, Chicago, IL (1995).
8. R. Hahn and R. Noll, Barriers to implementing tradable air pollution permits: Problems of regulatory interactions, *Yale J. Regul.* **1**, 63–91 (1983).
9. L. M. Hartman and D. Seastone, "Water Transfers: Economic Efficiency and Alternative Institutions," Johns Hopkins Press, Baltimore, MD (1970).
10. R. N. Johnson, M. Gisser, and M. Werner, The definition of a surface water right and transferability, *J. Law Econom.* **24**, 273–288 (1981).
11. J. O. Ledyard and K. Szakaly-Moore, Designing organizations for trading pollution rights, *J. Econom. Behavior Org.* **25**, 167–196 (1994).
12. M. H. Lennihan, The California drought emergency water bank: A successful institutional response to severe drought, *in* "Water Law: Trends, Policies and Practice" (K. M. Carr and J. D. Crammond, Eds.), American Bar Association, Section of Natural Resources, Energy and Environmental Law, Chicago, IL (1995).
13. D. Leonard and N. V. Long, "Optimal Control Theory and Static Optimization in Economics," Cambridge Univ. Press, Cambridge, MA (1992).
14. A. McGartland and W. Oates, Marketable permits for the prevention of environmental deterioration, *J. Environ. Econom. Management* **12**, 207–228 (1985).
15. W. Montgomery, Markets in licenses and efficient pollution control programs, *J. Econom. Theory* **5**, 395–418 (1972).
16. N. Spulber and A. Sabbaghi, "Economics of Water Resources: From Regulation to Privatization," Kluwer Academic, Boston, MA (1994).
17. A. Tarlock, Reallocation: It really is here, *in* "Water Law: Trends, Policies and Practice" (K. M. Carr and J. D. Crammond, Eds.), American Bar Association, Section of Natural Resources, Energy and Environmental Law, Chicago, IL (1995).
18. R. D. C. Wong and J. W. Eheart, Market simulations for irrigation water rights: A hypothetical case, *Water Resour. Res.* **19**, 1127–1138 (1983).