Journal of Environmental Economics and Management **37**, 85–105 (1999)
Article ID jeem.1998.1053, available online at http://www.idealibrary.com on IDEAL

A Dual Approach to Modeling Corner Solutions in Recreation Demand

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Received September 23, 1997; revised September 2, 1998

The dual approach of Lee and Pitt to estimating demand systems for which individuals often choose not to consume one or more of the available goods provides a utility-consistent framework for estimating preferences over visits to recreation sites. Because of the complexity of the model, however, this approach has not been applied in the recreation demand literature. This paper provides the first application of the model to recreation demand and develops a methodology for conducting welfare analysis, relying on Monte Carlo integration to derive estimates of compensating variation. The methods are applied to the demand for fishing in the Wisconsin Great Lakes region. © 1999 Academic Press

1. INTRODUCTION

A typical data set for analyzing recreation demand includes information on the total number of trips made by individuals during a season as well as the breakdown of the number of trips made to each of several available sites. Very rarely does the individual visit all of the available sites; instead *corner solutions* dominate, where individuals visit a subset of the sites multiple times, setting their demand for the other sites to zero. Neither continuous demand models nor discrete choice models alone have proved completely adept at analyzing data of this variety. A model is needed that addresses both the total number of trips made and the allocation of these trips to available sites.

The travel cost literature has largely adopted the approach first suggested by Bockstael et al. [1, 2] for solving the corner solution problem. Their method has subsequently been modified and applied by Hausman et al. [15], Parsons and Kealy [26], Feather et al. [9], Creel and Loomis [3], and Yen and Adamowicz [35], among others. Although these works differ slightly, they share the basic model design of examining the recreation decision in two steps. In the first step the random utility framework is employed to determine the allocation of trips based on characteristics and costs of reaching the sites. In the second step the total number of trips is estimated using a regression of trips on individual characteristics and a preference-weighted index, computed from the results of the first step. A combination of results from the two steps is used for welfare analysis.

A second strategy for dealing with corner solutions takes a more structural or behavioral approach. Based on work by Wales and Woodland [34], it begins with the maximization of a random utility function. The standard Kuhn–Tucker conditions are then also random variables and form the basis for probabilistic statements

0095-0696/99 $30.00

regarding when corner conditions will occur and for constructing the likelihood function. The method has been extended subsequently to a dual form by Lee and Pitt [19], starting with the specification of an indirect utility function. This approach is theoretically equivalent to the Kuhn–Tucker model and uses the concept of virtual prices to identify corner solutions. The dual model has the advantage of allowing the use of flexible-form indirect utility functions such as the translog.

The appeal of the Kuhn–Tucker and dual approaches lies in the unified and utility consistent framework they provide for characterizing corner solutions. Morey et al. [25], in a review of corner solution approaches in recreation demand, suggest that the Kuhn–Tucker/dual approach is preferable when estimation is feasible, given its consistency with utility theory. Because of their complexity, however, there have been only a few applications in the consumer choice literature of either model (e.g. [20, 30, 32, 34]). Phaneuf et al. (PKH) [28] provide an application of the Kuhn–Tucker approach to the recreation demand literature, modeling the demand for fishing in the Wisconsin Great Lakes region.[1] The purpose of this paper is to present an initial application of Lee and Pitt's dual approach to characterizing the demand for recreation and estimate welfare changes. The dual approach provides an opportunity to use a more general utility function and error structure specification than the Kuhn–Tucker approach. A methodology is applied for estimating compensating variation in the context of the dual model, employing Monte Carlo integration to derive expected welfare measures for changes in characteristics of fishing sites. The same data set used by PKH is used here as well, and a comparison of the results is presented.

2. THE DUAL APPROACH

A dual approach to modeling corner solutions beginning with statement of an indirect utility function was first suggested by Lee and Pitt [19]. Using their notation, the indirect utility function is defined as

$$H(\mathbf{v}; \theta, \boldsymbol{\varepsilon}) = \max_{\mathbf{q}} \{U(\mathbf{q}; \theta, \boldsymbol{\varepsilon}) \mid \mathbf{vq} = 1\}, \tag{1}$$

where $U(\cdot)$ is a strictly quasi-concave utility function, $\mathbf{q} = (q_1, \ldots, q_M)'$ is a vector representing the goods being analyzed, $\mathbf{v} = (v_1, \ldots, v_M)'$ is a vector of commodity prices normalized by income or product category expenditure (if weak separability is assumed), θ is a vector of utility function parameters, and $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_M)'$ is a vector of stochastic error terms. Application of Roy's Identity allows the recovery of *notional demands*:

$$q_i = \frac{\partial H(\mathbf{v}; \theta, \boldsymbol{\varepsilon})}{\partial v_i} \Bigg/ \partial v_i \sum_{j=1}^{M} v_j \frac{\partial H(\mathbf{v}; \theta, \boldsymbol{\varepsilon})}{\partial v_j}, \qquad i = 1, \ldots, M.^2 \tag{2}$$

[1] Bockstael et al. [2] describe the Kuhn–Tucker model within the context of recreation demand and propose the empirical specification used by Phaneuf et al.

[2] The unusual looking format is the result of the income normalization. That this is correct is apparent when we note that the derivative in the numerator is the result of differentiation with respect to the nonnormalized price, and the sum of the derivatives in the denominator results from the income term entering into each normalized price.

The q_i's are considered notional because they may take negative values, since the original problem in (1) does not include nonnegativity constraints. Thus $\mathbf{q}$ is meaningless economically; its elements should be interpreted rather as latent variables corresponding to observed demand $\mathbf{x} = (x_1, \ldots, x_M)'$ via the concept of virtual price. A virtual price is a reservation price that will exactly support zero consumption of a good. Similar to a tobit model, the virtual price is used in the formation of *actual demands* to transfer the probability mass associated with negative shares to the feasible region. For example, if the demands for the first k goods are observed to be zero, a vector of virtual prices $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_k)'$ supporting zero consumption can be solved using Roy's Identity from the equations

$$0 = \frac{\partial H(\pi_1(\bar{\mathbf{v}}), \ldots, \pi_k(\bar{\mathbf{v}}), \bar{\mathbf{v}}, \theta, \boldsymbol{\varepsilon})}{\partial v_i}, \qquad i = 1, \ldots, k, \tag{3}$$

where $\bar{\mathbf{v}}$ is the vector of prices for the positively consumed goods. Substituting the virtual prices for the nonconsumed goods into (2) yields demand equations for the $M - k$ consumed goods

$$x_i = \frac{\partial H(\pi_1(\bar{\mathbf{v}}), \ldots, \pi_k(\bar{\mathbf{v}}), \bar{\mathbf{v}}, \theta, \boldsymbol{\varepsilon})}{\partial v_i} \Bigg/ \sum_{j=1}^{M} v_j \frac{\partial H(\pi_1(\bar{\mathbf{v}}), \ldots, \pi_k(\bar{\mathbf{v}}), \bar{\mathbf{v}}, \theta, \boldsymbol{\varepsilon})}{\partial v_j}, \qquad i = k + 1, \ldots, M. \tag{4}$$

Selection of the subset of goods to be consumed, known as the demand regime, is determined by comparison of the virtual and actual prices. If the market price is higher than the virtual price, the good will not be consumed. The regime for which the first k goods are not consumed is characterized by

$$\pi_i(\bar{\mathbf{v}}) \leq v_i, \qquad i = 1, \ldots, k. \tag{5}$$

Equations (4) and (5) are used to state the probability of a particular consumption pattern, from which a likelihood function can be derived and the parameters of the indirect utility function recovered. This is developed explicitly in the context of the specific behavioral model below.

An intuitive explanation of the use of virtual prices in a two-good model is provided in Fig. 1.[3] The utility-maximizing observed consumption bundle in this case is a corner solution, where $x_1 = 0$ at market prices (p_1, p_2). If the utility function were maximized without regard to nonnegativity constraints, the solution would be the notional demands (q_1, q_2), where the first good is consumed at a negative quantity. The virtual price π_1 for the first good is a reservation price at which consumption of the good is induced to be exactly zero. By using the price ratio π_1/p_2 rather than p_1/p_2, we are able to "manufacture" a tangency condition for the observed consumption bundle, which can be used to form an estimating equation. We also note, in the case of a corner solution, that the market price is greater than the virtual price of the nonconsumed good. Comparison of the virtual

[3] Srinivasan [31] presents a similar diagram to explain corner solutions.

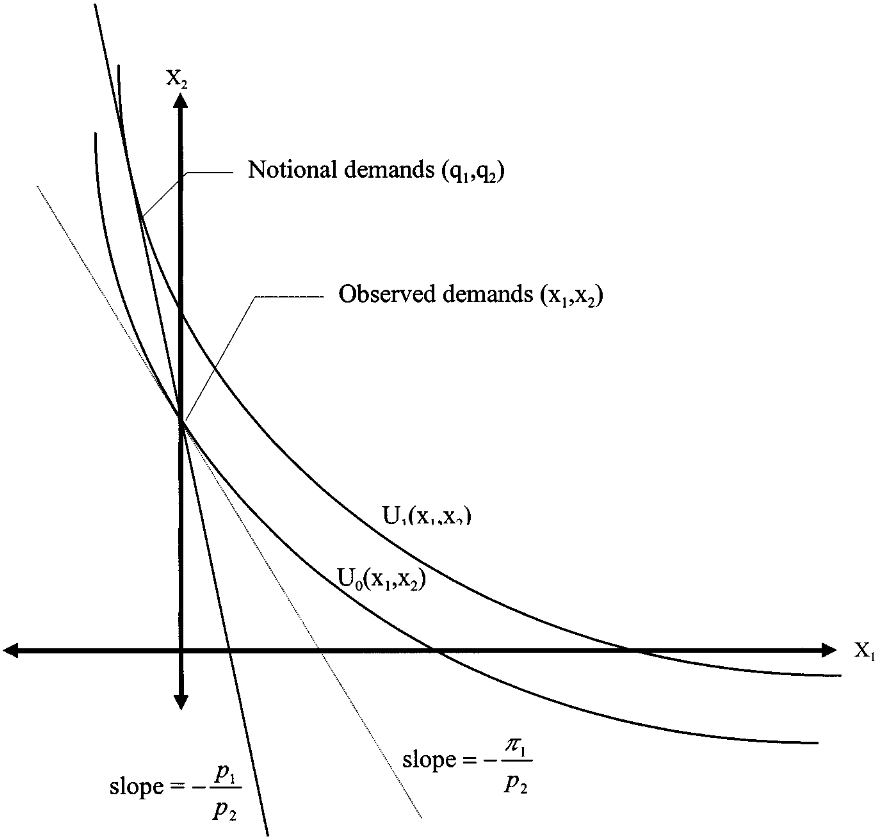

FIG. 1. Example of corner solution and virtual price.

price with the market price can therefore be used to identify which goods are nonconsumed.

3. MODEL SPECIFICATION

3.1. *Behavioral Model*

We begin by assuming preferences for trips to M recreation sites can be represented using duality theory by a random indirect utility function. Furthermore, it is assumed that the indirect utility function is weakly separable in the recreation goods. This implies a two-stage utility maximization process, where in the first stage the individual chooses expenditure on recreation and all other goods, and in the second stage the recreation expenditures are allocated among the available sites. The indirect utility function under this assumption is specified as a function of a subutility function for recreation goods, taking the form $VF(\mathbf{p}, y) = VF(V(\mathbf{p}_r, y_r), \mathbf{p}_a, y_a)$, where $V(\cdot)$ is the recreation subutility function, $\mathbf{p}_r$ and y_r are prices and expenditures for recreation goods, and $\mathbf{p}_a$ and y_a are prices and expenditures for all other goods. In this paper we will focus our attention on estimating the second stage of the two-stage process, i.e., the allocation of recreation expenditures among the available recreation sites.[4]

Following [19] and [32], the recreation indirect subutility function is represented using a version of the translog indirect utility function. That is,

$$\ln V(\mathbf{p}(y), \gamma, \boldsymbol{\varepsilon}) = \alpha_0 + \sum_{i=1}^{M} \alpha_i \ln p_i + \tfrac{1}{2} \sum_{i=i}^{M} \sum_{j=1}^{M} \beta_{ij} \ln p_i \ln p_j + \sum_{i=1}^{M} \varepsilon_i \ln p_i, \quad (6)$$

where $V(\cdot)$ is indirect subutility, $\mathbf{p} = (p_1, \dots, p_M)'$ is a vector of trip prices (round-trip travel cost plus opportunity cost of travel time) normalized by total recreation expenditures y, $\boldsymbol{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_M)'$ is a vector of error terms, and $\gamma = (\boldsymbol{\alpha}, \mathbf{B})$ are parameters of the utility function, where $\boldsymbol{\alpha} = (\alpha_0, \dots, \alpha_M)'$, $\mathbf{B} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M)'$, and $\boldsymbol{\beta}_j = (\beta_{j1}, \dots, \beta_{jM})$. It is standard practice with the use of the translog function (Christensen et al. [4]) to assume equality and symmetry restrictions on the matrix $\mathbf{B}$. That is,

$$\sum_{i=1}^{M} \beta_{ij} = \sum_{i=1}^{M} \beta_{ik}, \qquad j, k = 1, \dots, M, \quad (7a)$$

$$\beta_{ij} = \beta_{ji}, \qquad i, j \colon i \neq j. \quad (7b)$$

For the current application, additional structure is assumed on the translog function. The equality assumption is strengthened to

$$\sum_{i=1}^{M} \beta_{ij} = \sum_{j=1}^{M} \beta_{ij} = 0, \quad (7a')$$

and the error terms are restricted such that

$$\sum_{i=1}^{M} \varepsilon_i = 0. \quad (8)$$

The restriction in Eq. (7a′) is also used in previous applications of the dual model and implies that the indirect utility function is homogeneous [4, 20, 32]. Necessary for model tractability, this is a somewhat restrictive assumption in that it implies homotheticity of the utility function, which in turn implies that the expenditure terms drop out of the notional demand equations. In a standard application of a translog system of demands, this is consistent with the restrictive notion that expenditure elasticities are equal for all goods and all individuals in the sample. In the case of the dual model, however, the use of virtual prices in constructing the actual share equations will allow income effects to enter demand. This will be seen in the derivation of the actual share equations below.

[4] Although not without its drawbacks, the assumption of weak separability is common in applied demand studies. Edgerton [7, 8] discusses weak separability in the general context of demand estimation, as well as for the specific case of estimation of expenditure share models for which group expenditure enters nonlinearly. LaFrance [18] provides a careful overview of the ramifications for applied welfare analysis, concluding that the assumption will tend to bias welfare measures. The ramifications of the assumptions of weak separability and predetermined recreation expenditures for the current study will be further noted in later sections.

It is desirable for welfare analysis purposes to include in the model variables describing the characteristics of the recreation sites. Within the context of the translog function it is convenient to define the α_i's as functions of site quality variables. Assuming a linear relationship, we define a *quality index* for each site as

$$\alpha_i = -\left(d_i + \sum_k \delta_k q_{ki}\right), \qquad i = 1, \dots, M,$$

where $\mathbf{q}_i = (q_{1i}, \dots, q_{ki})'$ is a vector of quality variables for the ith site and the d_i's and δ_k's are parameters. Because the expenditure shares must sum to one, a normalization is necessary. As in [13], the restriction

$$\sum_{i=1}^{M} d_i = 1 \tag{10}$$

is employed.

Application of the logarithmic form of Roy's Identity to (6) and enforcement of the restrictions in (7) and (9) provides expressions for the notional expenditure shares

$$s_i = p_i q_i = \left(-d_i - \sum_k \delta_k q_{ki} + \sum_{j=1}^{M} \beta_{ij} \ln p_j + \varepsilon_i\right) \Big/ A, \qquad i = 1, \dots, M, \tag{11}$$

where

$$A = \sum_{l=1}^{M} \alpha_l.$$

To ensure the existence of the notional shares, it is necessary to define the quality indices such that the sum of the α_i's is either strictly positive or strictly negative. If the q_{ki}'s are assumed to positively influence utility (i.e., $\delta_k > 0$), then Eqs. (9) and (10) imply $A < 0$, which ensures that the share equations exist for all individuals in the sample. It is also then the case that the terms $-\delta_k q_{ki}/A$ enter the share equations positively.[5]

Defining $A < 0$ also allows an unambiguous interpretation of the β_{ij} parameters in the notional share equations. Note that β_{ij}/A is the own or cross-price notional effect of the jth price on the ith good. From the signs of the estimated β_{ij} coefficients we can infer something of the substitutability between the recreation sites.

3.2. *Derivation of Estimating Equations*

As noted above, the estimation process is derived using virtual prices. Consider the case in which the first k sites are not visited. Recalling that the virtual price is a type of reservation price that induces zero consumption, we can solve for the logs

[5] It is not generally necessary that the quality variables influence utility positively. If the quality index is to contain variables that are assumed to affect utility negatively, the α's can be defined using an alternative functional form such as the exponential. This would allow negative effects while enforcing the restriction that the sum of α's be strictly positive or negative.

of the reservation prices $\ln \boldsymbol{\pi}_{\mathbf{k}} = (\ln \pi_1, \ldots, \ln \pi_k)'$ by replacing $\ln p_j$ with $\ln \pi_j$, $j = 1, \ldots, k$, in the first k equations in (11), setting the shares equal to zero, and solving simultaneously. This is equivalent to solving for $\ln \boldsymbol{\pi}_{\mathbf{k}}$ in the system of equations

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 + \sum_{j=1}^{k} \beta_{1j} \ln \pi_j + \sum_{j=k+1}^{M} \beta_{1j} \ln p_j + \varepsilon_1 \\ \vdots \\ \alpha_k + \sum_{j=1}^{k} \beta_{kj} \ln \pi_j + \sum_{j=k+1}^{M} \beta_{kj} \ln p_j + \varepsilon_k \end{bmatrix}. \tag{12}$$

This yields solutions for the virtual prices:

$$\begin{bmatrix} \ln \pi_1 \\ \vdots \\ \ln \pi_k \end{bmatrix} = -\mathbf{B}_k^{-1} \begin{bmatrix} \alpha_1 + \sum_{j=k+1}^{M} \beta_{1j} \ln p_j \\ \vdots \\ \alpha_k + \sum_{j=k+1}^{M} \beta_{kj} \ln p_j \end{bmatrix} - \mathbf{B}_k^{-1} \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}, \tag{13}$$

where

$$\mathbf{B}_k = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1k} \\ \vdots & \ddots & \vdots \\ \beta_{k1} & \cdots & \beta_{kk} \end{bmatrix}.$$

As was intuitively shown in Fig. 1, for a good to be nonconsumed it must be the case that the good's market price exceeds its virtual price. For the case in which the first k goods are not consumed, it must therefore be that

$$\begin{bmatrix} \ln p_1 \\ \vdots \\ \ln p_k \end{bmatrix} \geq \begin{bmatrix} \ln \pi_1 \\ \vdots \\ \ln \pi_k \end{bmatrix}; \tag{14}$$

i.e., all of the market prices must be greater than the virtual prices for the nonconsumed goods. Substituting the expression in (13) for the virtual prices in (14) and manipulating algebraically, this relationship can be conveniently expressed as

$$\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \geq \begin{bmatrix} t_1 \\ \vdots \\ t_k \end{bmatrix}, \tag{15}$$

where

$$\begin{bmatrix} t_1 \\ \vdots \\ t_k \end{bmatrix} = -\begin{bmatrix} \alpha_1 + \sum_{j=k+1}^{M} \beta_{1j} \ln p_j \\ \vdots \\ \alpha_k + \sum_{j=k+1}^{M} \beta_{kj} \ln p_j \end{bmatrix} - \mathbf{B}_k \begin{bmatrix} \ln p_1 \\ \vdots \\ \ln p_k \end{bmatrix}.$$

It is now possible to state share equations for the remaining, positively consumed goods. Once again recalling Fig. 1, the rationale for using virtual prices was to "manufacture" a tangency solution for estimation. This was accomplished by substituting the virtual price for the actual price of the nonconsumed good. To derive share equations for $M - k - 1$ of the remaining goods ($M - k - 1$ equations are used since the last share is not independently identified), the virtual prices from (13) are substituted into the remaining share equations in (11) and set equal to the observed expenditure shares s_i. This yields

$$s_i = \left[\alpha_i + \sum_{j=1}^{K} \beta_{ij} \ln \pi_j + \sum_{j=k+1}^{M} \beta_{ij} \ln p_j + \varepsilon_i \right] \Big/ A, \qquad i = k+1, \ldots, M-1, \tag{16}$$

where A is as defined above. For derivation of the estimating equations it is convenient to separate the stochastic and nonstochastic terms in (16) and to rewrite the share equations as

$$s_i = \left[\alpha_i + \sum_{j=1}^{K} \beta_{ij} \ln \bar{\pi}_j + \sum_{j=k+1}^{M} \beta_{ij} \ln p_j + \varepsilon_i + \eta_i \right] \Big/ A, \qquad i = k+1, \ldots, M-1, \tag{17}$$

where

$$\eta_i = (\boldsymbol{\beta}_k^i)(\mathbf{B}_k^{-1}) \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix},$$

$$\boldsymbol{\beta}_k^i = (\beta_{i1}, \ldots, \beta_{ik}),$$

and $\ln \bar{\pi}_j$ is the deterministic component of the virtual price in (13).[6] Note that η_i contains the stochastic components of the virtual prices in (13). Rearranging (17) gives

$$\varepsilon_i = t_i, \qquad i = k+1, \ldots, M-1, \tag{18}$$

[6] It can be seen from a careful examination of Eq. (17) and the virtual prices in (13) that in the specification of actual demand expenditure does not drop out of the share equation, with the exception of the all goods consumed case, for which actual demand is equal to notional demand. Regime-specific site expenditure elasticities can be computed that are not equal for all individuals and sites.

where

$$t_i = s_i A - \left[\alpha_i + \sum_{j=1}^{k} \beta_{ij} \ln \overline{\pi}_j + \sum_{j=k+1}^{M} \beta_{ij} \ln p_j + \eta_i \right].$$

Equations (15) and (18), along with the specification of the joint density function $f_{\boldsymbol{\varepsilon}}(\boldsymbol{\varepsilon})$ for $\boldsymbol{\varepsilon}$, provide the necessary information for constructing the likelihood function for estimation. The contribution to the likelihood function for an individual who visits $M - k$ of the available sites is given by

$$\int_{t_1}^{\infty} \cdots \int_{t_k}^{\infty} g(\varepsilon_1, \ldots, \varepsilon_k) h(\varepsilon_{k+1} = t_{k+1}, \ldots, \varepsilon_{M-1} = t_{M-1} \mid \varepsilon_1, \ldots, \varepsilon_k)$$
$$\times |A^{M-k-1}| \, d\varepsilon_1 \cdots d\varepsilon_k, \qquad (19)$$

where $g(\cdot)$ is a marginal distribution and $h(\cdot)$ is a distribution conditional on the error terms appearing in the definition of η_i in Eq. (17). The $|A^{M-k-1}|$ term is the relatively simple Jacobian transformation from $\boldsymbol{\varepsilon}$ to $(\varepsilon_1, \ldots, \varepsilon_k, s_{k+1}, \ldots, s_{M-1})'$. There are 2^{M-1} possible consumption regimes for which a probability such as (19) can be constructed. The likelihood function is formed as the product of the appropriate probabilities, and maximum likelihood is used to recover estimates of the indirect utility function parameters.[7]

3.3. *Welfare Analysis Methodology*

The primary purpose for estimating corner solution models of recreation demand by the dual method is to provide an internally consistent, utility theoretic platform from which to conduct welfare analysis. We will be interested in the compensating variation associated with changes in site quality variables or the elimination of a site. Let the indirect subutility function from Eq. (6) be redenoted $V(\mathbf{p}, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon})$ to make explicit the presence of recreation expenditure y and site quality variables $\mathbf{q}$ in the function. The conditional compensating variation (C) associated with a change in the price and/or quality vectors from $(\mathbf{p}^0, \mathbf{q}^0)$ to $(\mathbf{p}^1, \mathbf{q}^1)$ is implicitly defined by[8]

$$V(\mathbf{p}^0, y, \mathbf{q}^0, \gamma, \boldsymbol{\varepsilon}) = V(\mathbf{p}^1, y + C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon}), \mathbf{q}^1, \gamma, \boldsymbol{\varepsilon}). \qquad (20)$$

Two attributes of the compensating variation measure are worthy of note. First, from the analyst's perspective, $C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon})$ is a random variable. We are therefore typically interested in the average value in the population, $\overline{C}$. Second, the nonlinear nature of the utility function will preclude a closed form for C. Numerical techniques will be required for its computation.[9]

PKH suggest a methodology for computing compensating variation within the context of the Kuhn–Tucker model. A modified version of this approach is

[7] Phaneuf [27] provides additional details on the derivation of the likelihood function.

[8] The compensating variation is conditional on the initial choice of recreation expenditure from the first stage of the utility maximization process.

[9] This problem has recently been addressed in nonlinear site selection models by McFadden [24] and Herriges and Kling [16].

employed here. Recall that the indirect utility function (6) is computed without nonnegativity constraints, allowing the possibility of negative expenditure shares. Economically meaningful shares are obtained using virtual prices, from which conditional indirect utility functions can be constructed for each possible demand regime. The indirect utility function of interest for welfare analysis is the maximum of the set of conditional indirect utility functions. Formally, let

$$D = \{\{1\}, \dots, \{M\}, \{1,2\}, \{1,3\}, \dots, \{1,2,\dots,M\}\} \tag{21}$$

denote the collection of all possible subsets of the index set $I = \{1, \dots, M\}$, each representing a possible demand regime. A conditional indirect utility function can then be defined for each $\omega \in D$ as

$$V_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon}) = V(\mathbf{p}_\omega, \pi(\mathbf{p}_\omega), y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon}), \tag{22}$$

where the commodities indexed by ω are consumed, $\mathbf{p}_\omega = \{p_j\colon j \in \omega\}$, and $\pi(\mathbf{p}_\omega)$ is the vector of virtual prices for the nonconsumed goods. Application of the logarithmic form of Roy's Identity to $V_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon})$ yields conditional share equations $\mathbf{s}_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon})$, the utility maximizing consumption levels for the given regime. Note that both V_ω and $\mathbf{s}_\omega$ are functions of $\mathbf{q}$ and not $\mathbf{q}_\omega = \{q_j\colon j \in \omega\}$, since the choice of indirect utility function in Eq. (6) does not exhibit the property of weak complementarity [22]. This implies that compensating variation will contain both use and nonuse value components.[10] The presence of nonuse values in the estimates of compensating variation will be further addressed below.

Constraining a subset of the commodities to zero via virtual prices provides no assurance that the shares for the remaining goods will be positive. Let

$$\tilde{D} = \left\{\omega \in D\colon s_{\omega j}(\mathbf{p}_\omega, y; \mathbf{q}, \gamma, \boldsymbol{\varepsilon}) > 0\, \forall j \in D\right\} \tag{23}$$

denote the set of ω's for which the corresponding conditional indirect utility function yields nonnegative shares. The nonnotional indirect utility function of interest for welfare analysis is then the maximum of the feasible conditional indirect utility functions. That is,

$$V(\mathbf{p}, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon}) = \max_{\omega \in \tilde{D}} \{V_\omega(\mathbf{p}_\omega, y, \mathbf{q}, \gamma, \boldsymbol{\varepsilon})\}. \tag{24}$$

The computation of compensating variation in Eq. (18) corresponds to implicitly solving for $C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon})$ in

$$\max_{\omega \in \tilde{D}^0} \left\{V_\omega(\mathbf{p}^0_\omega, y; \mathbf{q}^0, \gamma, \boldsymbol{\varepsilon})\right\} = \max_{\omega \in \tilde{D}^1} \left\{V_\omega(\mathbf{p}^1_\omega, y + C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, \gamma, \boldsymbol{\varepsilon}); \mathbf{q}^1, \gamma, \boldsymbol{\varepsilon})\right\}. \tag{25}$$

In practice there are three difficulties associated with computing $\overline{C}$. First, for a given $\boldsymbol{\varepsilon}$ and $\gamma, C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon})$ is an implicit function for which no closed

[10] The absence of weak complementarity implies that individuals may assign "nonuse" value to the resource in addition to "use value"; i.e., the individual receives utility from the availability of the good without actually consuming it. Freeman's [10] definitions of use value, nonuse value, and existence value are applied here.

form exists. A numerical procedure such as numerical bisection must be employed. Second, given $C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon})$ and γ, $\overline{C}$ does not have a closed-form solution, requiring the use of Monte Carlo methods to evaluate. Errors can be drawn from the underlying distribution for $\boldsymbol{\varepsilon}$, $f_{\varepsilon}(\boldsymbol{\varepsilon})$, and the average of the resulting $C(\mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, y, \gamma, \boldsymbol{\varepsilon})$'s forms an estimate of $\overline{C}$.[11] Third, given an algorithm for computing $\overline{C}$, the analyst does not typically have available γ, but rather an estimator $\hat{\gamma} \sim g_{\hat{\gamma}}$. Thus any computation of $\overline{C}$ will itself be a random variable, dependent upon the distribution of the estimated parameters. The procedure developed by Krinksy and Robb [17] can be employed to approximate the statistical properties of $\hat{C}$, the estimate of $\overline{C}$, by repeatedly drawing realizations from $g_{\hat{\gamma}}$ and computing $\hat{C}$ for each of these realizations.

4. DATA

The empirical application of the dual model focuses on angling in the Wisconsin Great Lakes region. The data are drawn from two mail surveys of angling behavior conducted in 1990 at the University of Wisconsin–Madison.[12] The surveys provide detailed information on the 1989 angling behavior of Wisconsin fishing license holders, including the number and destination of fishing trips to the Wisconsin Great Lakes region, the distances to each destination, the type of angling preferred, and socioeconomic characteristics of the survey respondents. Of the 509 completed surveys, 266 individuals visited the Great Lakes during 1989 and are the focus of this study.

While the survey provides information on 22 distinct Great Lakes fishing destinations, these destinations are combined into four aggregate "sites" for this application:

- Site 1: South Lake Michigan
- Site 2: North Lake Michigan
- Site 3: Green Bay
- Site 4: Lake Superior

This aggregation divides the Wisconsin portion of the Great Lakes into distinct geographical zones, consistent with the Wisconsin Department of Natural Resources (WDNR) classification of the lakes region.

The price of a single trip to each of the four fishing sites consists of two components: the travel cost and the opportunity cost of travel time. Round-trip travel costs were computed for each destination and each individual by multiplying the number of round-trip miles for a given individual–destination combination by the cost per mile for the vehicle class driven, as provided by the American Automobile Association. The proper means of computing the opportunity cost of travel time is less straightforward. McConnell and Strand [23] suggest in an often-cited paper using one-third of the individual's wage rate to approximate the opportunity cost of an hour of travel time. Implicit in this assumption is that income represents a full potential income constraint, including the value of time,

[11] Geweke [12] provides a useful review of Monte Carlo methods.
[12] Details of the survey and sampling procedure are available in Lyke [21].

from which leisure can be traded for money. Most studies employing this measure of the opportunity cost of time, however, include earned income in the model. Because the emphasis of this paper is on dealing with corner solutions, rather than modeling time costs, this relatively rough approach to measuring time costs will be used here as well. For each site, travel time costs are computed using one-third of the wage rate and assuming an average travel speed of 45 miles per hour. The price of visiting a destination is the sum of the direct travel costs and the cost of travel time.

Two types of attribute variables are used to characterize the quality of the recreation sites: fishing catch rates and an indicator of boat ownership. Catch rates are important since anticipated success of fishing is likely to be a major determinant in the recreation decision. Furthermore, state and federal agencies currently spend large amounts of time and money to influence catch rates in the region through stocking programs and regulations. The use of catch rates in the model will allow the model to be used to conduct welfare analyses of existing or alternative fishery management programs.

In constructing the catch rate variables, attention is focused on the success rates for the four aggressively managed salmonid species: lake trout, rainbow (steelhead) trout, Coho salmon, and Chinook salmon. Creel surveys by the WDNR provide 1989 catch rates for each of these species at each of the 22 disaggregate destinations used in the surveys. Each of these catch rates is broken down by angling method, including private boat, charter fishing, and pier/shore fishing. Data from the survey were used to match the mode-specific catch rates to each fisher based upon their most frequent mode of fishing. Catch rate creel surveys are conducted each year by the WDNR and are independent of the survey data used in the study. A primary advantage of using these exogenous catch rate data is that we avoid the endogeneity problems associated with using internally generated catch rates, as discussed in [6].

A site catch rate index for use in the model

$$CAT_i = R_{lk,i} + R_{ch,i} + R_{co,i} + R_{rb,i}, \qquad i = 1, \dots, 4, \tag{26}$$

was constructed for each site, where $R_{k,i}$ denotes the catch rate for species k at site i, $k = lk$ for lake trout, ch for Chinook salmon, co for Coho salmon, and rb for rainbow trout.

An indicator of boat ownership is included in the attribute variables, since ownership of a boat suitable for the Great Lakes use is likely to affect how the lakes can be used and therefore the recreation decision. An indicator variable B is constructed, where $B = 1$ if the individual owns a Great Lakes-suitable boat, and $B = 0$ otherwise. With catch rates and the boat indicator included as attribute variables (along with the site-specific intercept term), the site quality parameter α_i can be defined as in Eq. (9) as

$$\alpha_i = -(d_i + \delta_0 B + \delta_1 CAT_i), \qquad i = 1, \dots, 4. \tag{27}$$

Tables I and II provide summary statistics for the data. Table I focuses on the mean and standard deviation for usage, price, and attribute variables for the four sites. Table II characterizes the distribution of sites visited among the survey respondents. Note that many of the visitors visit only one site (72%), but a substantial percentage (28%) visit multiple sites, stressing the necessity of using a method that effectively deals with corner solutions.

TABLE I
Average Site Characteristics

	South Lake Michigan	North Lake Michigan	Green Bay	Lake Superior
1989 fishing trips	4.50	2.99	1.25	5.27
	(11.96)	(8.51)	(4.17)	(18.09)
Price	93.04	123.97	128.65	163.83
	(101.75)	(112.41)	(109.65)	(123.36)
Catch rate index	0.17	0.144	0.062	0.1452
	(0.134)	(0.087)	(0.051)	(0.085)

Notes: Standard deviations are in parentheses. Catch rates are measured as fish per person-hour of effort. Of the 266 individuals in the sample, 99 indicated ownership of a boat.

5. RESULTS

5.1. *Model Estimation*

For estimation it is assumed that $\boldsymbol{\varepsilon} \sim N(0, \sigma^2\Sigma)$, where $\Sigma = \{r_{ij}\}$, and r_{ij} denotes the correlation between ε_i and ε_j. Since the disturbance terms are constrained to sum to zero, Srinivasan and Winer [32] show that the correlation coefficients can be specified by the constraint

$$\sum_{\substack{j=1 \\ i<j}}^{M-1} r_{ij} = \frac{2-M}{2}, \tag{28}$$

where the left-hand side is the sum of correlations between all pairs of errors for the first $M-1$ errors. This implies that there are $[(M-1)(M-2)/2] - 1$ free

TABLE II
Distribution of Trips

Sites visited	No. of observations
All four sites, $\omega = \{1, 2, 3, 4\}$	3
South and North Lake Michigan and Green Bay, $\omega = \{1, 2, 3\}$	13
South and North Lake Michigan and Lake Superior, $\omega = \{1, 2, 4\}$	1
South Lake Michigan, Green Bay, and Lake Superior, $\omega = \{1, 3, 4\}$	0
North Lake Michigan Green Bay, and Lake Superior, $\omega = \{2, 3, 4\}$	7
South and North Lake Michigan, $\omega = \{1, 2\}$	13
South Lake Michigan and Green Bay, $\omega = \{1, 3\}$	4
South Lake Michigan and Lake Superior, $\omega = \{1, 4\}$	8
North Lake Michigan and Green Bay, $\omega = \{2, 3\}$	19
North Lake Michigan and Lake Superior, $\omega = \{2, 4\}$	10
Green Bay and Lake Superior, $\omega = \{3, 4\}$	2
South Lake Michigan, $\omega = \{1\}$	85
North Lake Michigan, $\omega = \{2\}$	46
Green Bay, $\omega = \{3\}$	11
Lake Superior, $\omega = \{4\}$	49

TABLE III
Parameter Estimates

Variable	Parameter	Estimate	P-value
Intercept (North Lake Michigan)	d_2	1.78	< 0.001
Intercept (Green Bay)	d_3	0.14	0.278
Intercept (Lake Superior)	d_4	−0.21	0.276
Boat	δ_0	0.37	0.002
Catch rate index	δ_1	1.88	0.008
	β_{11}	0.29	0.027
	β_{22}	0.56	0.005
Elements of matrix **B**	β_{33}	0.22	0.0615
	β_{44}	1.32	< 0.001
	β_{23}	0.14	0.104
	β_{12}	−0.13	0.163
Standard deviation of $\boldsymbol{\varepsilon}$	σ	1.43	< 0.001

Notes: The intercept term for South Lake Michigan is $(1 - d_2 - d_3 - d_4)$. The remaining elements of **B** can be computed as functions of the estimated elements.

parameters to be estimated in a completely general specification of Σ, in addition to the variance σ^2 [31]. To simplify the estimation process, the additional assumption of exchangeability of the ε's is maintained. The exchangeability assumption can be thought of as a weaker alternative to independence, since correlation among the errors is allowed. Under exchangeability (see [11] or [5] for a more complete description) it is assumed that the joint distribution of $\boldsymbol{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_M)'$ is invariant to permutations of the indexes $(1, \dots, M)$. This assumption implies that Σ has the structure

$$\Sigma = \begin{bmatrix} 1 & r & r & r \\ r & 1 & r & r \\ r & r & 1 & r \\ r & r & r & 1 \end{bmatrix}, \tag{29}$$

i.e., $r_{ij} = r \; \forall i \neq j$. Furthermore, given that the errors sum to zero, it can be shown from Eq. (28) that $r = -1/3$ for a four-good model. The correlation matrix of interest for estimation is therefore

$$\Sigma = \begin{bmatrix} 1 & -1/3 & -1/3 & -1/3 \\ -1/3 & 1 & -1/3 & -1/3 \\ -1/3 & -1/3 & 1 & -1/3 \\ -1/3 & -1/3 & -1/3 & 1 \end{bmatrix}. \tag{29'}$$

Given the above assumptions on the error structure, maximum likelihood was used to recover parameter estimates of the indirect utility function, presented in Table III.[13] The signs of the estimated coefficients are generally significant and are

[13] A variety of starting values were used in numerous runs of the estimation program. As expected, given the highly nonlinear likelihood function, the parameter estimates are somewhat sensitive to starting values. The reported estimates are those for which the highest likelihood value was achieved, given that model coherency conditions were met.

of the expected signs. The estimate for the boat indicator variable (δ_0) is positive and significant, indicating that ownership of a Great Lakes-suitable boat affects the demand for trips positively. As expected, the catch rate index parameter (δ_1) is also positive, indicating that increases in catch rates affect utility and the demand for trips positively. We therefore expect that fishery management policies or other actions that decrease catch rates will have a negative effect on consumer welfare. This will be further addressed in the welfare section below. In addition to the quality variable parameters, estimates of three of the site-specific intercept terms are presented. The intercept term for South Lake Michigan can be computed from the estimates for the other three sites. Given that the intercept terms sum to one, and the quality variables are shown to influence utility positively, we are assured that the sum of the quality index terms (the α_i's) is strictly negative and the share equations are defined for all individuals in the sample.

The estimates of the elements of the price-effects matrix **B** were also generally as expected. The restriction in Eq. (7) implies that there are six free parameters in **B**. Estimates of the four on-diagonal elements and two of the off-diagonal elements are presented, from which the remaining elements can be computed as functions of these according the model restrictions. The subscripts on the elements of the price effects matrix correspond to the site numbers from the previous section. Van Soest and Kooreman [33] note that a sufficient (although not necessary) condition for model coherency (the sum of the probabilities of each demand regime adding up to one), given the restrictions in the model, is that **B** be positive semidefinite, which the estimated parameters meet. The matrix of marginal own and cross log-price effects on notional shares is given by $\mathbf{B}/A$. An estimate of this, computed using the estimated mean of the sum of the quality indices, is presented in Table IV.

Since these estimates represent effects on notional expenditure shares, interpretation of the magnitude of these numbers is not economically interesting. The signs of the estimated parameters do have intuitive value, however. All own price effects are negative, as would be expected. With two exceptions, all cross-price terms are positive, implying that the goods are substitutes, as is expected when modeling recreation goods that are closely related in their characteristics. The positive cross-price term between North Lake Michigan and Green Bay implies these two goods have a share complementary relationship. This means that if the prices of all of the other goods rise, the consumption shares of both North Lake Michigan and Green Bay will increase. Since prices of the goods are based on travel distance and Green Bay and North Lake Michigan are geographically close, implying highly

TABLE IV

Estimates of Own and Cross-Price Marginal Effects on Notional Shares

	South Lake Michigan	North Lake Michigan	Green Bay	Lake Superior
South Lake Michigan	−0.115	0.051	−0.043	0.107
North Lake Michigan	0.051	−0.221	−0.0544	0.225
Green Bay	−0.043	−0.054	−0.089	0.186
Lake Superior	0.107	0.225	0.186	−0.519

Note: Computed as $\mathbf{B}/sample\ mean\ (\Sigma_{i=1}^{M} \hat{\alpha}_i)$, where $\hat{\alpha}_i = -(\hat{d}_i + \hat{\delta}_0 B + \hat{\delta}_1 CAT_i)$.

correlated prices, the sign of the North Lake Michigan/Green Bay price was expected. The nonintuitive sign on the South Lake Michigan/Green Bay term, however, was not expected and may be due in part to the relatively small number of data points used to estimate a complicated system. A larger number of observations would provide more precision in these estimates.

The estimates of the price–effects matrix and quality indices can also be used to compute own-price, cross-price, and expenditure elasticities for the sites, conditional on the observed demand regime. Previous applications of the dual and Kuhn–Tucker models (e.g. [20, 29]) have focused on providing such estimates. Of greater interest in recreation demand, however, are the welfare effects of changing characteristics of the recreation sites. The estimated model is used in the following section to address this.

5.2. *Welfare Analysis*

A primary advantage of the dual approach to corner solution analysis is that it permits the construction of welfare measures from an internally consistent and utility-theoretic framework. In this subsection the estimated dual model in Table III and the methodology introduced above are used to evaluate the welfare effects of a series of policy scenarios for the Wisconsin Great Lakes region.

The Great Lakes region provides many opportunities for policy-relevant welfare experiments, as the lakes are heavily managed. Large population areas surround parts of the lakes, causing environmental concerns. The fishery itself is in many ways artificially created and maintained. Of the major species included in the model, only lake trout are native to both Lake Superior and Lake Michigan. Rainbow trout, Coho salmon, and Chinook salmon were introduced during the twentieth century through stocking programs. Although these species now reproduce naturally in the lakes, their populations are heavily augmented through aggressive stocking programs. The lakes have also been invaded by exotic species, including the sea lamprey, which preys on lake trout. Accidentally introduced in 1930s, the sea lamprey decimated lake trout populations in the lakes. A breeding population has been successfully reintroduced into Lake Superior, while a fishable population in Lake Michigan is maintained only through stocking programs. Expensive sea lamprey control efforts continue to this day in the lakes. Given these examples of intervention in the management of the Great Lakes, it is natural to ask if the benefits of these programs offset the corresponding costs. The dual model can be used to measure program benefits. To illustrate this use, welfare effects are estimated under four hypothetical policy scenarios:

- *Scenario A: Loss of Lake Michigan lake trout.* State and local efforts to stock lake trout in the three Lake Michigan sites are eliminated, driving the catch rate for lake trout to zero at these sites.

- *Scenario B: Reduction of Lake Michigan Coho salmon.* State and local efforts to stock Coho salmon in the three Lake Michigan sites are eliminated, leaving only the breeding population to sustain the fishery. It is assumed for scenario purposes that catch rates at the three sites are half of their former level.

- *Scenario C: Increase in Lake Michigan rainbow trout.* Increased state and local efforts to stock rainbow trout in the three Lake Michigan sites lead to a 20% increase in catch rates at the three sites.

TABLE V
Welfare Estimates

Policy scenario	Mean (use-only) compensating variation
Scenario A: Loss of lake trout in Lake Michigan sites	203.21
Scenario B: Reduction of Coho salmon in Lake Michigan sites	107.90
Scenario C: Increase in rainbow trout in Lake Michigan	−6.30
Scenario D: Loss of South Lake Michigan site	1962.26

- *Scenario D: Loss of South Lake Michigan site.* Because of changes in pollution control and environmental quality standards in the surrounding population centers, South Lake Michigan is no longer suitable for recreational fishing. Catch rates for all species are set to zero, and the price is set infinitely high.

For each of these scenarios, the average compensating variation in the population of Great Lakes anglers $\overline{C}$ was estimated by the following procedure:

- For each observation in the sample ($n = 1, \ldots, 266$) a total of $N_\varepsilon = 1000$ vectors of random disturbances terms (i.e., $\varepsilon^{(nk)}$, $k = 1, \ldots, N_\varepsilon$) were drawn from the estimated distribution for $\boldsymbol{\varepsilon}$.

- Substituting the maximum likelihood estimates of the parameters $\hat{\gamma}$ and $\varepsilon^{(nk)}$ for γ and $\boldsymbol{\varepsilon}$ in Eq. (25), numerical bisection was used to solve for C, with the result labeled $C^{(nk)}$.

- Taking the mean of the $C^{(nk)}$'s over the N_ε disturbance vectors and N observations in the sample yields a point estimate $\hat{C}$ for average compensating variation in the population, given $\hat{\gamma}$.

An additional step can be added to the calculation to account for the fact that both use values and nonuse values are present in the estimates of compensating variation. Nonuse values, as defined by Freeman [10], arise in this model from the fact that catch rates for the nonvisited sites enter the demands for the visited sites through the virtual prices. Policymakers are often most interested in the use values associated with the resource, since it is this value that can be most clearly defined for policy purposes. To eliminate the nonuse value in the calculation of compensating variation, in each draw of the error in the Monte Carlo process the virtual prices for the nonvisited sites can be computed assuming catch rates for those sites are zero. In this way the demands for the visited sites are functions of only the catch rates for the visited sites, implying that welfare calculations will reflect only the use value associated with the resource.

Point estimates of the mean compensating variation for each of the four experiments are presented in Table V. Our attention focuses on estimates of the use-only component of the welfare measure, since these are usually of most interest to policymakers.[14] The welfare effects for scenarios A, B, and D are positive, since they are willing to accept figures for damage to the resource, while scenario C is negative, representing a willingness to pay for an improvement in the

[14] Total compensating variation measures were also computed. The use-only values reported here consist of one-half to three-fourths of the total value.

resource. The magnitudes of the estimates are generally consistent with prior expectations, based on the results presented in PKH. The loss of an entire site near the region's population center has a predictably high mean welfare estimate of approximately $1960 per angler per year, while the loss of half the Coho salmon in Lake Michigan produces a mean loss of $107.90. Mean willingness to pay for a 20% increase in rainbow trout in Lake Michigan is estimated as −$6.30. This figure is low in magnitude compared to the other estimates, since in assuming that recreation expenditures are predetermined for estimation purposes, we constrain the willingness to pay estimates by the recreation budget. It is in this figure that the restrictive nature of this assumption is apparent, since in a fully general model willingness to pay would be constrained by total income rather than the recreation budget. The lake trout estimate of approximately $203 would seem to justify the effort that has gone into rehabilitating the fishery during the past three decades. Although the magnitude of this estimate is plausible, it contradicts the findings of PKH, who report an insignificant welfare loss for the lake trout scenario. It is interesting to note that the result reported here is derived using a catch rate index (to reduce the number of parameters to be estimated), which does not allow the model to differentiate preferences for the various species of sport fish, while PKH use separate catch rates in their estimates. This highlights the importance of model specification decisions in the magnitude of estimated welfare effects.

As described in Section 3 it is possible to use the methodology suggested by Krinsky and Robb [17] to compute confidence intervals for the welfare measures. This procedure is not carried out in the current study, since the small number of observations used do not provide enough precision to compute confidence intervals that are policy-relevant.

6. SUMMARY AND CONCLUSIONS

This study has provided an initial empirical application of Lee and Pitt's dual approach to the problem of corner solutions in recreation demand, estimating the demand for fishing in the Wisconsin Great Lakes region as well as welfare measures associated with changes in Great Lakes attributes. Use of the dual method is appealing in that it approaches corner solutions in a manner consistent with behavioral theory, providing a solid theoretical framework for the resulting welfare measures. It also allows the use of flexible form utility functions and general error structures.

The results of the application were consistent with prior expectations. In general the estimated coefficients were significantly different from zero and of the expected sign. The estimated parameters satisfy the model coherency requirement without having to impose additional restrictions. Likewise, the welfare estimates are of the expected sign and general magnitudes for the relatively extreme policy scenarios examined. The estimates compare well with the findings of PKH, when consideration is given to the different specification for quality indices used in the two studies and the bias associated with the assumptions of weak separability and predetermined recreation expenditures.

There are many opportunities to expand on and improve the work presented here. It would be desirable to relax the assumption of weak separability and include a numeraire good, allowing substitution between spending on recreation and other goods, as well as incorporating the effects of nonusers of the resource. Most importantly, this would eliminate the bias in the welfare measures attributable to weak separability and predetermined expenditures, as described by LaFrance [18]. It would also allow a more direct comparison with PKH, and would in turn allow one to determine the robustness of their results to the choice of utility function and error distribution. Such comparisons are valuable in that if estimates from simple utility functions are similar to those resulting from more complicated utility functions, their use may be more justifiable. Finally, more observations would be used, decreasing the variance of the estimated parameters and increasing the precision of welfare calculations. In this case it may be worthwhile to pursue the Krinsky and Robb procedure, or a standard bootstrap, to calculate standard errors for the welfare effects. Should it prove intractable to estimate the model including a numeraire good, a second best approach may be to endogenize recreation expenditure by separately estimating a recreation expenditure equation (the first stage of the two-stage process), in the spirit of Goddard and Amuah [13]. It would then be necessary to modify the welfare measurement method to account for the possibility of changes in recreation expenditures resulting from changes in site attributes.

Econometrically, the model can be improved by relaxing some of the restrictions used in this presentation. For example, the homotheticity assumption could be relaxed. This would involve estimating more elements of the price effects matrix **B** and would require a larger data set than was available for this study to achieve significant results. In addition, the exchangeability assumption could be relaxed and the full correlation matrix estimated. The challenges associated with this generalization are similar to those associated with estimating multinomial probit models. Promising new simulation methods (see [14] for a review) for estimating multidimensional normal integral probabilities, however, appear to be on the verge of general applicability and should be useful in further generalizations of the dual model.

Structurally there are possibilities to improve the model as well. Use of species-specific catch rates, as opposed to a catch rate index, would allow the welfare effects to reflect differing angler preferences for species. Most importantly, the opportunity cost of time should be further addressed. For this initial application the simplest assumption of one-third of the wage rate has been used. It would be interesting, however, to expand the model to include allocation of time decisions. With further research that addresses these points, optimism that the dual approach has the potential to provide a merging of a second-order approximation utility function, normal errors, and a utility-consistent model for the analysis of corner solutions in recreation demand seems warranted.

ACKNOWLEDGMENTS

Helpful comments on earlier versions of this paper were provided by Catherine Kling, Joseph Herriges, Noel Cressie, and two anonymous referees. Thanks are also owed to Audrey Lyke, Richard Bishop, and Mike Welsh, who generously shared the data used in this study with me.

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