Measurement Error in Recreation Demand Models: The Joint Estimation of Participation, Site Choice, and Site Characteristics*

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In the demand for recreational fishing sites, an important explanatory variable differentiating sites is the unobserved expected catch rate. Since the observed catch rate is subject to sampling variability, using the average of a site's observed catch rates causes the parameter estimator on catch to be biased downward. We develop and demonstrate a solution to this errors-in-variables problem when there are repeated measurements on the catch rate. Consistent and efficient estimates of both the demand parameters and the expected catch rates are obtained by simultaneously estimating them by maximum likelihood. An empirical example demonstrates the importance of simultaneous estimation. © 1998 Academic Press

1. INTRODUCTION

The use value recreational anglers receive from our nation's rivers and streams are determined in a major way by catch rates and the influence of those catch rates on demand and utility (e.g., [12, 9, 28, 30]). Impacts from mining, industry, and other sources injure fish stocks, which can affect catch rates. Examples include oil spills [3, 5, 9], chemical spills [14], soil erosion and nonpoint agricultural pollution [25], the release of hazardous substances from mining and mineral processing [20], dams [21], and acid rain [22].

Accurate estimation of how much such injuries damage society is important: the dollar estimates of damages and cleanup determine whether cleanup is efficient, and under CERCLA the polluter is, in general, legally responsible for all past and residual damages. In this paper we show that the literature has consistently underestimated the importance of catch rates, and thus underestimated the damages associated with reductions in catch rates. The reason previous studies have underestimated the catch rate is that they have used the average of observed catch rates, rather than the true (but unobserved) catch rate as a regressor. This errors-in-variables problem results in attenuation bias. We propose and implement a solution to the problem.

Consider modeling where and how often an individual fishes. An important determinant of these choices is the expected catch rates at the available sites. The expected catch rate is defined as the catch rate at a site for a typical angler on a typical day. It is a characteristic of a site, such as acreage or the availability of boat ramps. But unlike those characteristics, expected catch rates are not observed.

Suppose the researcher has two data sets: a travel-cost data set and a catch data set. The travel-cost data set includes, for a group of individuals, how many trips each sampled individual took to each available site, along with individual-specific trip costs, income, and other individual-specific characteristics such as skill level, which can affect catch. The catch data set includes a set of observed catch rates for each site. Typically, the observed number of catch rates will vary significantly by site, with often more observed catch rates for sites that are thought to have high expected catch rates, and fewer observed catch rates for sites that are thought to have low expected catch rates.

The two data sets might or might not overlap. There is no overlap if the individuals who generated the catch data are not included in the travel-cost data set. There is complete overlap if the catch data is solely from the individuals in the travel-cost data set. A hybrid case is if some of the individuals who generated the catch data also appear in the travel-cost data set.[1]

The problem is to use these two data sets to estimate the parameter associated with expected catch rate in the demand function. Typically, this is done in two steps. An estimate of the expected catch rate at a site is obtained by averaging the observed individual catch rates for that site, and then these average catch rates are entered into the demand functions for the sites as if they were the expected catch rates measured without error. This practice is almost universal in models of recreational fishing demand that include catch rate as an explanatory variable. A few recent examples are [9, 10, 15, 21, 26, 27].[2]

However, since observed catch rates are subject to sampling variability, the average of the observed catch rates for a site is not the site's expected catch rate, but rather its expected catch rate measured with error. In addition, the degree of error varies by site, and is inversely related to the number of observed catch rates at the site. Consequently, an errors-in-variables problem exists, which, if not corrected, results in parameter estimates that are both biased and inconsistent. That is, with the standard two-step approach, the parameters that explain the influence on demand of expected catch and travel cost are both biased.

When there is only one observation on the variable measured with error (in this case, the expected catch rate for a site), the only solution to the errors-in-variables

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[1] In our empirical example, a travel-cost study done for the state of Montana [18, 20], catch data were collected for one trip from each of 1344 anglers. There were 443 (33% of the total) individuals followed for a season to determine how many trips each individual took to each available site. In this example, the travel-cost data consists of travel patterns for 443 individuals. Three different catch data sets could be constructed: catch data from the 901 individuals who are not in the travel-cost data set (the case of no overlap), catch data from only those 443 individuals in the travel-cost data set (complete overlap), and all 1344 observed catch rates in the catch data set (partial overlap).

[2] A few studies use stock size rather than catch rate as the measure of fishing quality. See, for example, [4]. A few studies, (e.g., [2, 3]) have used catch ratings such as those that appear in newspapers. McConnell *et al*. [15] allow expected catch to vary across anglers. They model an individual's total catch on a trip as the result of a Poisson process, which implies the estimated expected catch rate is expected catch measured with error. They separately estimate the Poisson catch process, and then use the resulting estimated catch rates as exogenous variables in their recreational demand model.

problem is the questionable method of instrumental variables.[3] When there is more than one observed catch rate for each site, consistent and efficient estimates of both expected catch rates and the parameters in the demand functions for the sites can be obtained by abandoning the two-stage approach mentioned above and using maximum likelihood to simultaneously estimate the expected catch rates and the parameters in the demand functions.

Intuitively, there are two types of information about the expected catch rates. First, there is the average of each site's observed catch rates, and, second, there is the observed trip patterns. *Ceteris paribus*, individuals take more trips to sites with high expected catch rates and fewer trips to sites with lower expected catch rates; observed trip patterns are therefore significant predictors of expected catch rates.

If data on individual catch rates were available, but not data on trip patterns, the best (minimum variance) statistical estimate of a site's expected catch rate would be the average of the site's observed catch rates. Alternatively, if there were data on trip patterns, but no data on individual catch rates, the best statistical estimates of expected catch rates would be those estimates that, along with trip costs and other explanatory variables, best explain the observed trip patterns. However, when data are available on both individual catch rates and individual trip patterns, the best statistical estimate of a site's expected catch rate is a weighted average of these two separate estimates, where the weight on the sample average catch rate is a decreasing function of the sampling variation in the observed catch rates relative to the sampling variation in the observed trip patterns. Such a weighting is critical because if the average catch rate is computed from a large number of observations (a popular site), that average is likely to be a good estimate of the site's expected catch rate, but the average catch rate will likely be a poor estimate of a site's expected catch rate if it is based on only a small number of observations.

This model was first proposed by Morey and Waldman [23] and Morey and Rowe [18] to explain choice of fishing sites in Montana. The model is new but has some similarities to the multiple-indicator–multiple-cause (MIMIC) model of an unobservable variable [11] and to the analytic technique of factor analysis [13] where, in our case, the unobservable is the true catch rate.[4] It differs from a MIMIC model in that we do not explain catch, and it differs from both models in that our indicators (observed catch) are more closely related to the unobserved variable (expected catch) than are indicators in a typical application of those models.

Although the application here is recreational fishing, the issues and theory apply to any empirical setting where an average is used as an explanatory variable: for example, human capital models that use the average of a number of IQ test scores, rather than unobserved IQ, as an explanatory variable, and salary models that use batting average, rather than the unobserved probability of getting a hit, as an explanatory variable (e.g., [29]).[5]

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[3]This method is not often implemented, however, because it is difficult to find an instrument that is correlated with explanatory variables but not correlated with the error, and because the parameter estimates are usually sensitive to the instrument chosen.

[4]Englin and Shonkwiler [7] consider cases where trip cost is unobserved, and propose a factor analytic model that uses time, out-of-pocket expenses, and value of time as indicators of trip cost.

[5]Note that in both of these examples, as with recreational fishing sites, the number of observations used to calculate each average will vary: individuals expected to have either high or low IQs take more IQ tests, and the batting averages for those with high expected hit rates are based on more times at bat than those of bad hitters.

There is a general trend in recreational demand modeling and the maximum likelihood estimation of these models to specify broader models and simultaneously estimate all of the parameters of those models.[6] While this research is part of that trend, it differs from it in that in most of these models piecemeal estimation is consistent but not asymptotically efficient. Therefore, joint estimation buys only asymptotic efficiency. In the situation analyzed here, two-stage estimators are biased and inconsistent, while joint estimation is consistent and efficient.

This paper is organized as follows: First, to fix ideas, we discuss estimation of a simple linear demand model for trips to several fishing sites. Demand for trips is modeled as a function of demographic and site-specific variables, as well as the site's expected catch rate, where the expected catch rate is not observed, but where there are multiple observed catch rates for each site. Then, discrete choice estimation with the same kinds of data is presented, first in the simple dichotomous setting, the probit model, and then in a repeated nested logit model. The final section considers an example with relevance to environmental policy.

2. THE LINEAR REGRESSION MODEL

Assume that the number of trips to site j for individual i, y_{ji}, is a linear function of a set of K known, exogenous site and individual attributes, including travel cost, site quality, etc., $\mathbf{x}_{ji}$, and the unknown expected catch rate at site j, c_j^*:[7]

$$y_{ji} = \boldsymbol{\alpha}' \mathbf{x}_{ji} + \gamma c_j^* + \epsilon_{ji}, \qquad j = 1, \ldots, n, i = 1, \ldots, m, \qquad (1)$$

where m is the number of anglers, γ and the elements of $\boldsymbol{\alpha}$ are unknown parameters to be estimated, and ϵ_{ji} is a mean-zero random disturbance with variance σ_ϵ^2.[8] For purposes of exposition and notation in this section, we have not explicitly included the attributes of other sites in (1), since their inclusion would not change the theoretical econometric results. Cross-site effects are included in the empirical example. If the c_j^* were known, linear regression of y_{ji} on $\mathbf{x}_{ji}$ and c_j^* would result in unbiased and consistent estimation of $\boldsymbol{\alpha}$ and γ.

Let $\mathbf{c}^* = (c_1^*, c_2^*, \ldots, c_n^*)'$ and let $\mathbf{X}$ be the $nm \times K$ matrix with typical row $\mathbf{x}_{ji}'$. Then the model expressed in matrix form is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \gamma \mathbf{D}\mathbf{c}^* + \boldsymbol{\epsilon}, \qquad (2)$$

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[6] For example, in the estimation of nested logit models there is increasing advocacy of FIML rather than sequential estimation. In recreational demand models, it is increasingly common to estimate determinants of travel cost (such as the value of time as a percentage of wage) along with the parameter on travel cost. See, for example, [20].

[7] The influence of expected catch on demand, γ, could, for example, be assumed to be a function of skill. In our empirical example, skill is a significant determinant of the influence of expected catch on participation and site choice.

[8] The model and analysis remain appropriate if c_j^* represents some other unobservable determinant of demand.

where $\mathbf{y} = (y_{11}, \ldots, y_{1m}, \ldots, y_{nm})'$, the $nm \times n$ matrix $\mathbf{D}$ is defined by

$$
\mathbf{D} = \begin{pmatrix}
\boldsymbol{\iota} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \boldsymbol{\iota} & \cdots & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\iota}
\end{pmatrix},
$$

and $\boldsymbol{\iota}$ is the $m \times 1$ vector of 1's. Let $\mathbf{W}_* = [\mathbf{X} \vdots \mathbf{D}\mathbf{c}^*]$, and let $\boldsymbol{\beta} = (\boldsymbol{\alpha}'\boldsymbol{\gamma})'$. Then (2) can be written as

$$
\mathbf{y} = \mathbf{W}_*\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{3}
$$

and least squares produces

$$
\hat{\boldsymbol{\beta}} = (\mathbf{W}'_*\mathbf{W}_*)^{-1}\mathbf{W}'_*\mathbf{y} = \boldsymbol{\beta} + (\mathbf{W}'_*\mathbf{W}_*)^{-1}\mathbf{W}'_*\boldsymbol{\epsilon}, \tag{4}
$$

where $\hat{\boldsymbol{\beta}} = (\hat{\boldsymbol{\alpha}}'\ \hat{\boldsymbol{\gamma}})'$ and the second equals sign is by substitution from (3). Since $\mathbf{c}^*$ and the columns of $\mathbf{X}$ are assumed to be exogenous, $E(\mathbf{W}'_*\boldsymbol{\epsilon}) = \mathbf{W}'_* E(\boldsymbol{\epsilon}) = \mathbf{0}$ so that $E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$.

But the c_j^* are not observed. Instead, suppose there are L_j reported catch rates for site j, c_{jl}. These observed catch rates are subject to the vagaries of fishing, so they are random variables. Assume each c_{jl} is an unbiased estimate of the true catch rate:

$$
c_{jl} = c_j^* + \delta_{jl}, \qquad j = 1, \ldots, n, l = 1, \ldots, L_j, \tag{5}
$$

where δ_{jl} are mean-zero random disturbances with variance σ_δ^2.[9] It is assumed that the δ_{jl} are uncorrelated with each of the ϵ_{ji}. This is an implicit assumption whenever the average catch is used as an explanatory variable in a travel-cost model. There is no question of corrrelation when the two data sets do not overlap, as errors are not generated by the same decision-making units. For data sets that

[9]More generally observed catch rate could be modeled as a function of skill, site experience, weather, etc. This would be in the spirit of the MIMIC model of Joreskog and Goldberger [11].

Englin *et al.* [6] also jointly estimate catch rate and the catch parameter, but in a fundamentally different model. Specifically, they specify catch as a function of variables that affect catch rate but not number of trips:

$$
c_j^* = \boldsymbol{\lambda}'\mathbf{z}_j + \delta_j, \qquad j = 1, \ldots, n, \tag{5'}
$$

where c_j^* is total catch and $\mathbf{z}_j$ is a set of environmental and angler-specific causes of c_j^*. Note that (5') implies that c_j^* is a random variable, but it is specified as parametric in the travel-cost component of their model. This approach requires data that affect catch and not travel behavior (to achieve identification of all model parameters), and requires the δ_j and model disturbances to be uncorrelated.

In Englin and Shonkwiler [7], price, rather than catch, is the variable measured with error. In lieu of repeated measures of price, their solution to the measurement error is a factor analysis model where $p_{j1} = $ cost, $p_{j2} = $ travel time, and $p_{j3} = $ wage $\times$ travel time are used as *indicators* of price. Equation (5) is replaced by

$$
p_{jl} = \lambda_j p_j^* + \delta_{jl}, \qquad j = 1, \ldots, m, l = 1, 2, 3. \tag{5''}
$$

Equation (5''), while interesting, does not follow naturally from the standard assumption that inputs, such as time and transportation services, are combined to produce trips, and that p_j^* is the minimum cost of producing a trip. Note that since the p_{jl} are only loosely associated with p_j^*, the coefficients λ_j cannot be normalized to 1 as in our model.

do overlap, ϵ_{ji} and the δ_{jl} are uncorrelated if fishing skill is not site specific, that is, if skilled anglers are skilled at all sites and low-skilled anglers are low skilled at all sites. Even if skill is site specific, ϵ_{ji} and the δ_{jl} will remain uncorrelated if the influence of expected catch is made a function of the individual's skill at the site.

Denote the average of site j's observed catch rates by $\bar{c}_j$, where

$$\bar{c}_j = \frac{1}{L_j} \sum_{l=1}^{L_j} c_{jl},$$

and let $\bar{\mathbf{c}} = (\bar{c}_1, \ldots, \bar{c}_n)'$. Note that $E(\bar{\mathbf{c}}) = \mathbf{c}^*$. Let $\mathbf{W} = [\mathbf{X} \vdots \mathbf{D}\bar{\mathbf{c}}]$. A common estimation strategy is to use $\bar{\mathbf{c}}$ instead of $\mathbf{c}^*$ in (2). To investigate the implications of this, replace $\mathbf{c}^*$ in (2) with $\bar{\mathbf{c}}$:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \gamma\mathbf{D}\bar{\mathbf{c}} + \mathbf{v} = \mathbf{W}\boldsymbol{\beta} + \mathbf{v}, \tag{6}$$

where $\mathbf{v} = \boldsymbol{\epsilon} + \gamma\mathbf{D}(\mathbf{c}^* - \bar{\mathbf{c}})$. Then least squares produces

$$\mathbf{b} = (\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{y} = \boldsymbol{\beta} + (\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{v}. \tag{7}$$

It is clear that $\mathbf{b} = (\mathbf{a}'g)'$, where $\mathbf{a}$ is an estimator of $\boldsymbol{\alpha}$ and g is an estimator of γ, is *not* an unbiased estimator of $\boldsymbol{\beta}$ since $\mathbf{W}$ and $\mathbf{v}$ are clearly correlated through their common term $\bar{\mathbf{c}}$. It is easy to show that $E(g) < \gamma$, when catch rate is the only variable measured with error.[10] The noise in average catch rate makes catch rate appear less important than it is as a determinant of trip choice. All studies that use $\bar{\mathbf{c}}$ as a proxy for $\mathbf{c}^*$, *ceteris paribus*, underestimate the influence of catch on participation and site choice, and therefore underestimate willingness to pay for policies that improve catch.

The analysis of (1) through (7) is all very reminiscent of the conventional errors-in-variables model (EVM), with two important differences. First, in the EVM, the expected value of each observation on the variable measured with error is an additional parameter, called a *nuisance* parameter, and here that role is played by the c_j^*. With an EVM, the number of parameters increases with the sample size, and there are more parameters $(n + K + 1)$ than observations (n). In the case at hand since there are *repeated* observations on c_j^*, there are more observations than parameters, so estimation is possible. And second, these often-called nuisance parameters in our case are important for demand prediction and consumer welfare analysis.

3. MAXIMUM LIKELIHOOD ESTIMATION

Define the vectors $\mathbf{z}_j' = (\mathbf{y}_j, \mathbf{c}_j)$, $j = 1, \ldots, n$, where $\mathbf{y}_j = (y_{j1}, \ldots, y_{jm})'$ and $\mathbf{c}_j = (c_{i1}, \ldots, c_{iL_j})'$. Define $\boldsymbol{\theta}' = (\boldsymbol{\beta}, \sigma_\epsilon^2, \sigma_\delta^2)$. By assumption, the independence of ϵ_{ji} and δ_{jl} implies the independence of y_{ji} and c_{jl}. Therefore the density of $\mathbf{z}_j$ can be factored:

$$f(\mathbf{z}_j) = f(\mathbf{y}_j) \times f(\mathbf{c}_j), \tag{8}$$

[10] See [8, pp. 439–440].

where $f(\cdot)$ are density functions of $\mathbf{z}_j$, $\mathbf{y}_j$, and $\mathbf{c}_j$, determined by the argument of the function. If, for example, normality is assumed,[11]

$$f(y_{ji}) = \frac{1}{\sqrt{2\pi}\,\sigma_\epsilon}\exp\left\{-\frac{1}{2\sigma_\epsilon^2}(y_{ji} - \boldsymbol{\beta}'\mathbf{w}_{ji}^*)^2\right\} = \sigma_\epsilon^{-1}\phi\left(\frac{y_{ji} - \boldsymbol{\beta}'\mathbf{w}_{ji}^*}{\sigma_\epsilon}\right),$$

$$f(c_{jl}) = \frac{1}{\sqrt{2\pi}\,\sigma_\delta}\exp\left\{-\frac{1}{2\sigma_\delta^2}(c_{jl} - c_j^*)^2\right\} = \sigma_\delta^{-1}\phi\left(\frac{c_{jl} - c_j^*}{\sigma_\delta}\right),$$

$$(9)$$

where w_{ji}^* is the row of $\mathbf{W}^*$ corresponding to y_{ji} and $\phi(a) = (1/\sqrt{2\pi})\exp(-1/2a^2)$ is the density of a standard normal random variable. Define $\mathbf{Z} = (\mathbf{z}_1, \ldots, \mathbf{z}_n)$. Then the log-likelihood may be written as

$$\log L\left(\mathbf{Z}|\boldsymbol{\beta}, \mathbf{c}^*, \sigma_\delta^2, \sigma_\epsilon^2\right)$$

$$= \sum_j \log f(\mathbf{z}_j) = \sum_j \left(\sum_i \log f(y_{ji}) + \sum_l \log f(c_{jl})\right)$$

$$= \sum_j \left(\sum_i \log\left\{\sigma_\epsilon^{-1}\phi\left(\frac{y_{ji} - \boldsymbol{\beta}'\mathbf{w}_{ji}^*}{\sigma_\epsilon}\right)\right\} + \sum_l \log\left\{\sigma_\delta^{-1}\phi\left(\frac{c_{jl} - c_j^*}{\sigma_\delta}\right)\right\}\right). \quad (10)$$

Insight into what this generalization of the model means toward the estimation of the expected catch rates and the marginal effect of catch rate on number of trips (γ) can be obtained by examining the first-order conditions for the maximization of the likelihood function with respect to $\boldsymbol{\alpha}$, γ, $\mathbf{c}^*$, σ_ϵ^2, and σ_δ^2:

$$\frac{\partial \log L}{\partial \boldsymbol{\alpha}} = \mathbf{0}, \Rightarrow \frac{1}{\sigma_\epsilon^2}\sum_j\sum_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji} - \gamma c_j^*)\mathbf{x}_{ji} = \mathbf{0}, \quad (11)$$

$$\frac{\partial \log L}{\partial \gamma} = 0 \Rightarrow \frac{1}{\sigma_\epsilon^2}\sum_j\sum_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji} - \gamma c_j^*)c_j^* = 0, \quad (12)$$

$$\frac{\partial \log L}{\partial c_j^*} = 0 \Rightarrow \frac{1}{\sigma_\epsilon^2}\sum_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji} - \gamma c_j^*)\gamma$$

$$+ \frac{1}{\sigma_\delta^2}\sum_l(c_{jl} - c_j^*) = 0, \qquad j = 1, \ldots, n, \quad (13)$$

and

$$\frac{\partial \log L}{\partial \sigma_\epsilon^2} = 0 \Rightarrow \sigma_\epsilon^2 = \frac{1}{n}\sum_j\frac{1}{L_j}\sum_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji} - \gamma c_j^*)^2, \quad (14)$$

$$\frac{\partial \log L}{\partial \sigma_\delta^2} = 0 \Rightarrow \sigma_\delta^2 = \frac{1}{n}\sum_j\frac{1}{L_j}\sum_l(c_{jl} - c_j^*)^2. \quad (15)$$

The maximum likelihood estimators are the values of all parameters that simultaneously solve (11) through (15) (the conventional estimator indicator "$\hat{\ }$," here and below, has been omitted for notational simplicity).

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[11] Normality is assumed here to facilitate the interpretation of the model. In the example in Section 5, the assumption of Poisson-distributed catch is employed.

From (12) by multiplying through by c_j^* and employing the linearity of the summation operators, the solution for γ is

$$\gamma = \frac{\Sigma_j(1/m)\Sigma_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji})c_j^*}{\Sigma_j c_j^{*\,2}}. \tag{16}$$

Substituting this into (13) yields the solution, after some algebra, for c_j^*:

$$c_j^* = \mu\bar{c}_j + (1 - \mu)\tilde{c}_j, \qquad j = 1,\ldots,n, \tag{17}$$

where

$$\mu = \frac{L_j}{L_j + m\lambda\gamma^2} \tag{18}$$

for $\lambda = \sigma_\delta^2/\sigma_\epsilon^2$, and

$$\tilde{c}_j = \frac{1}{m\gamma}\sum_i(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji}). \tag{19}$$

This $\tilde{c}_j$ is the fixed-effects panel data estimator of c_j^* obtained from only trip-taking behavior. This can be seen by noting that, apart from the presence of γ, (1) is the usual formulation of a panel data model with c_j^* taking the role of the group-specific effect. Note that the conventional estimator of c_j^*, $\bar{c}_j$, would involve catch data and not trip data, and would be solved from only the term after the plus sign in (13).

Since $0 \le \mu \le 1$, the ML estimator of c_j^* is a weighted average of the ML estimator of c_j^* in the absence of data on y_{ji} ($\bar{c}_j$), and the ML estimator of c_j^* in the absence of data on c_{jl} ($\tilde{c}_j$). The relative weights associated with $\bar{c}_j$ and $\tilde{c}_j$ reflect the confidence in the data on c_{jl} and y_{ji}. That is, $\lambda \to \infty \Rightarrow \mu \to 0$ and, consequently, all weight is placed on $\tilde{c}_j$. Intuitively, a large value of λ implies σ_ϵ^2 is small relative to σ_δ^2, so that there is relatively more noise in the c_{jl} data. Conversely, $\lambda \to 0 \Rightarrow \mu \to 1$ and, consequently, all weight is placed on $\bar{c}_j$, as a large σ_ϵ^2 relative to σ_δ^2 implies there is relatively more noise in the y_{ji} data. In fact, it is easy to show that the weights μ and $(1 - \mu)$ are the normalized reciprocals of the variances of the quantities they weight, $\bar{c}_j$ and $\tilde{c}_j$, making the ML estimator of c_j^* the optimally weighted linear combination of the two kinds of information (see [1, p. 130]). Finally, the ML estimator of γ is the least squares estimator of γ with the estimated value of c_j^* in place of the actual c_j^*.

A large-sample interpretation of the estimator of c_j^* is also possible. Suppose $L_j \to \infty$. Then $\mu \to 1$ and hence the ML estimator of c_j^* reduces to $\bar{c}_j$, which is consistent when $L_j \to \infty$. Next suppose that L_j is fixed and $m \to \infty$. Then clearly $\mu \to 0$ and hence the ML estimator of c_j^* reduces to $\tilde{c}_j$, which converges to $\gamma^{-1}E(y_{ji} - \boldsymbol{\alpha}'\mathbf{x}_{ji}) = c_j^*$ as $m \to \infty$. This contrasts with $\mathbf{b}$, the least squares estimator using $\bar{c}_j$ defined in (7), which is consistent in the sense that as $\min_j L_j \to \infty$, plim $L_j^{-1}\mathbf{W}'\mathbf{v} = \mathbf{0}$. However, the usual interest is in extending the sample size in the direction of m rather than L_j. Hence, for nonoverlapping data sets, least squares is obviously inconsistent when $m \to \infty$; that is, it is inconsistent even if the number of anglers in the travel-cost data set $\to \infty$. For overlapping data sets, $m \to \infty$ implies $L_j \to \infty$ for *some* j, but does not imply $\min_j L_j \to \infty$, so least squares remains inconsistent.

Note that a natural form of heteroscedasticity, unequal variances across sites, is easily accommodated in this setting. In obvious notation, σ_δ is replaced with σ_{δ_j} in the likelihood, and the first-order condition (15) becomes

$$\sigma_{\delta_j}^2 = \frac{1}{L_j} \sum_l (c_{jl} - c_j^*)^2. \tag{20}$$

In both the homoscedastic and this heteroscedastic model, average catch rates with larger variances receive less weight. With heteroscedasticity, a larger variance could be the result of *either* fewer observed catch rates or a larger site-specific variance, while with constant variance ($\sigma_{\delta_j}^2 = \sigma_\delta^2$ for all j) the influence of average catch rate depends only on L_j.

4. DISCRETE CHOICE MODELS

Suppose now that the model, for all j and i, is

$$y_{ji}^* = \boldsymbol{\alpha}' \mathbf{x}_{ji} + \gamma c_j^* + \epsilon_{ji}, \qquad j = 1, \ldots, n, \, i = 1, \ldots, m, \tag{21}$$

where y_{ji}^* is defined as the *net utility* of visiting site j. The difference between this model and the model above is that the y_{ji}^* are not observed. Instead, a dichotomous indicator of site choice is known:

$$y_{ji} = \begin{cases} 1 & \text{if } y_{ji}^* > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{22}$$

where y_{ji} is redefined such that $y_{ji} = 1$ if individual i visited site j during the season and $y_{ji} = 0$ otherwise. This model is designed to estimate fishing choice when the data set reports whether individual i visited site j but not how many trips he or she took.

This is the conventional probit model, given the normality assumption for ϵ_{ji}, except c_j^* is not observed.[12] The probability that $y_{ji} = 1$ is given by

$$\Pr(y_{ji} = 1) = \Pr(y_{ji}^* > 0) = \Pr(\epsilon_{ji} > -\boldsymbol{\alpha}' \mathbf{x}_{ji} - \gamma c_j^*)$$

$$= 1 - \Pr(\epsilon_{ji} < -\boldsymbol{\alpha}' \mathbf{x}_{ji} - \gamma c_j^*) = 1 - \Phi\left(-\frac{\boldsymbol{\alpha}' \mathbf{x}_{ji} + \gamma c_j^*}{\sigma_\epsilon}\right)$$

$$= \Phi\left(\frac{\boldsymbol{\alpha}' \mathbf{x}_{ji} + \gamma c_j^*}{\sigma_\epsilon}\right) \overset{\text{def}}{=} \Phi_j, \tag{23}$$

where $\Phi(a)$ is the cumulative distribution function of a standard normal random variable, and use has been made of its symmetry: $\Phi(a) = 1 - \Phi(-a)$. Of course, $\Pr(y_{ji} = 0) = 1 - \Pr(y_{ji} = 1)$. As in the linear regression model, c_{jl} is observed as an imperfect measure of c_j^* (see (5)). Now the likelihood is of the mixed

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[12] Morikawa *et al.* [24] consider unobserved explanatory variables in a binary probit model. Their model differs from ours in that, like Englin *et al.* [6], they assume the unobserved variable is a function of other variables and errors.

discrete/continuous type. With $\mathbf{z}_j$ and $\boldsymbol{\theta}$ defined as above, by the assumption of the independence of ϵ_{ji} and δ_{jl} the density of $\mathbf{z}_j$ can be factored:

$$f(\mathbf{z}_j) = \Pr(\mathbf{y}_j) \times f(c_j), \tag{24}$$

so that the log-likelihood is

$$\log L(\mathbf{Z}|\boldsymbol{\theta}, c_1^*, \ldots, c_n^*)$$

$$= \sum_j \left(\sum_i \log \Pr(y_{ji}) + \sum_l \log f(c_{jl}) \right)$$

$$= \sum_j \left(\sum_i \log[y_{ji} \Phi_j + (1 - y_{ji})(1 - \Phi_j)] + \sum_l \log \sigma_\delta^{-1} \phi \left(\frac{c_{jl} - c_j^*}{\sigma_\delta} \right) \right). \tag{25}$$

Maximization of the likelihood again produces consistent, asymptotically efficient estimates of $\boldsymbol{\theta}$ and the c_j^*.

Now consider a repeated nested logit model of participation and site choice that is designed to be estimated with a data set that reports, for the season, the number of trips each sampled individual takes to each of the n sites.[13] Assume the season consists of T periods such that in each period the individual takes at most one trip. In each period, the individual simultaneously decides both whether to fish at one of the n sites, and if so, which one.

Assume the utility individual i receives in period t if he chooses alternative j is

$$y_{jit}^* = \boldsymbol{\alpha}' \mathbf{x}_{jit} + \gamma c_j^* + \epsilon_{jit}, \qquad j = 0, \ldots, n, \tag{26}$$

where $j = 0$ is the nonfishing alternative and the expected catch rate for nonfishing is 0. Further assume that the $\boldsymbol{\epsilon}_{it} = (\epsilon_{0it}, \ldots, \epsilon_{nit})'$ are drawn independently in each period from the generalized extreme value distribution

$$F(\boldsymbol{\epsilon}_{it}) = \exp\left\{ -\exp(-\epsilon_{0it}) - \left[\sum_{j=1}^n \exp(s\epsilon_{jit}) \right]^{1/s} \right\}, \tag{27}$$

where s is a parameter that influences the degree of unobserved correlation between the utility from fishing trips. The y_{jit}^* are, of course, unobserved. The observed variable is y_{jit}, defined by

$$y_{jit} = \begin{cases} 1 & \text{if } y_{jit}^* > y_{j'it}^* \text{ for all } j' \neq j, \\ 0 & \text{otherwise}. \end{cases} \tag{28}$$

It is straightforward to show that

$$\Pr(y_{0it} = 1) = \frac{\exp(\boldsymbol{\alpha}' \mathbf{x}_{0it})}{\exp(\boldsymbol{\alpha}' \mathbf{x}_{0it}) + \left(\sum_{j=1}^n \exp\left[s(\boldsymbol{\alpha}' \mathbf{x}_{jit} + \gamma c_j^*) \right] \right)^{1/s}} \tag{29}$$

[13] The nested logit model was developed by McFadden [16]. Morey [17] provides a general introduction to repeated nested logit models of recreational participation and site choice.

and

$$\text{Pr}(y_{j'it} = 1) = \frac{\exp\big(s(\boldsymbol{\alpha}'\mathbf{x}_{j'it} + \gamma c_j^*)\big)\big(\sum_{j=1}^{n} \exp\big[s(\boldsymbol{\alpha}'\mathbf{x}_{jit} + \gamma c_j^*)\big]\big)^{1/s-1}}{\exp(\boldsymbol{\alpha}'\mathbf{x}_{0it}) + \big(\sum_{j=1}^{n} \exp\big[s(\boldsymbol{\alpha}'\mathbf{x}_{jit} + \gamma c_j^*)\big]\big)^{1/s}},$$

$$j' = 1, \ldots, n. \quad (30)$$

Note that since the expected catch rates are unobserved, they must be estimated along with the other parameters. Define $\mathbf{y}_i$ equal to an $(n + 1) \times 1$ vector, where the first element is the number of times individual i chooses not to fish and the remaining elements are the number of trips individual i takes to each of the n sites. That is, a typical element of $\mathbf{y}_i$ is $y_{ji} = \Sigma_t y_{jit}$. Let $\mathbf{Y} = (\mathbf{y}_1, \ldots, \mathbf{y}_m)$, and similarly define $\mathbf{X}$. Let $\Omega = (\mu_0, \boldsymbol{\beta}, \sigma_\delta^2, s)$. Then the likelihood function is

$$L(\mathbf{Y}, \mathbf{X} | \Omega, c_1^*, \ldots, c_n^*) = \sum_{j=0}^{n} \left(\sum_{i=1}^{m} \sum_{t=1}^{T} y_{ji} \log \text{Pr}(y_{jit}) + \sum_{l=1}^{L_j} \log \sigma_\delta^{-1} \phi\left(\frac{c_{jl} - c_j^*}{\sigma_\delta} \right) \right).$$

$$(31)$$

Expected catch rates and the parameter on catch in a repeated nested logit travel-cost model were jointly estimated by [31] assuming catch is normally distributed.

5. EMPIRICAL EXAMPLE: NATURAL RESOURCE DAMAGE ASSESSMENT IN MONTANA

The Montana study [20] demonstrates the importance of estimating the catch rates jointly with the other parameters in a travel-cost model, including the parameters on catch. A nested-logit model was used to estimate participation and site choice for 443 trout anglers. The choice set included 26 river segments, listed in the first column of Table I.

Trip catch and number of hours fished were collected from 1344 trips (each trip is by a different angler) to the 26 sites, where 443 of these trips are by the 443 individuals in the travel-cost data set, so there is 33% overlap between the two data sets. The data on catch and hours fished are summarized in columns 2–4 of Table I. The sites are listed in descending order of the total number of hours for which catch data are available. In spite of the fact that the interviewers spent approximately equal time at each site recording fishing times and catch, the number of trips for which there is catch data varies from 172 (at Madison 2) to 8 (at Little Blackfoot). The total number of hours of fishing for which catch is recorded varies even more, from 847.13 (at Madison 2) to 7.48 (at Warm Springs Creek). Such significant variation in hours and/or trip for which catch is observed is expected and should be a characteristic of most data sets on catch; there is more fishing at sites with high catch rates than at sites with low catch rates, so there are more hours of fishing to observe. There are no fish in Silver Bow Creek, and no one was observed fishing there. It is included in the choice set because it would be an excellent trout stream in the absence of environmental injury.

Since catch is nonnegative integer data, another, perhaps superior stochastic model for catch is the Poisson distribution. This was the assumption for catch data

used in [20]. A Poisson distribution for recreational catch was first suggested by [15] (see also [6]). Specifically, assume that catch has a Poisson distribution such that the probability of catching η_{jl} fish in h_{jl} hours of fishing is given by

$$\Pr(\eta_{jl}) = \frac{\exp(-h_{jl}c_j^*)(h_{jl}c_j^*)^{\eta_{jl}}}{\eta_{jl}!}. \tag{32}$$

With a random sample on η_{jl}, $l = 1, \ldots, L_j$, one estimate of a site's catch rate is the ratio of the total number of fish caught at that site to the total number of hours fished. This simple Poisson average catch rate is the maximum likelihood estimate of c_j^* in the absence of other data, and is reported in column 4 of Table I.[14] However, as explained above, this is not the best estimate of the catch rate when trip data are available. Using the trip and catch data for the 443 anglers, the 26 catch rates were jointly estimated with the other parameters in the travel-cost model, including the parameters on catch, travel costs, and other site characteristics. These included site size, measures of accessibility, aesthetics, and availability

[14] Note that the likelihood function is the final additive term in Eq. (31).

TABLE I
Catch Data and Catch Rate Estimates

River Segment	Observed number of trips	Number of fish caught	Total observed hours of fishing	Simple Poisson catch rate* (rank)	Expected catch rate** (rank)
Madison 2	172	615	847.13	0.7260 (12)	0.7260 (13)
Beaverhead	91	294	464.38	0.6331 (15)	0.6366 (15)
Rock Creek	109	394	406.37	0.9696 (5)	0.9262 (2)
Yellowstone	69	129	367.47	0.3511 (20)	0.3817 (20)
Missouri	102	232	345.45	0.6716 (14)	0.7604 (10)
Madison 1	83	132	341.48	0.3865 (18)	0.4128 (19)
Big Hole 1	77	265	314.65	0.8422 (9)	0.8985 (3)
Bitterroot 2	59	163	222.02	0.7342 (11)	0.7391 (11)
Jefferson 2	50	59	200.35	0.2945 (24)	0.3259 (21)
Big Hole 2	29	137	156.78	0.8738 (8)	0.8116 (7)
Middle Clark Fork	58	78	150.80	0.5172 (16)	0.4465 (16)
Gallatin	59	130	148.18	0.8773 (7)	0.8842 (4)
Jefferson 1	41	44	140.15	0.3140 (23)	0.2801 (22)
Upper Clark Fork 1	58	45	116.65	0.3858 (19)	0.4340 (17)
Blackfoot	50	32	97.28	0.3289 (21)	0.2672 (23)
Bitterroot 1	39	25	79.12	0.3160 (22)	0.2398 (24)
Upper Clark Fork 5	30	58	71.45	0.8118 (10)	0.7838 (8)
East Gallatin	41	98	66.33	1.4774 (2)	1.0510 (1)
Flint Creek	53	62	65.28	0.9497 (6)	0.8767 (5)
Upper Clark Fork 3	18	20	43.77	0.4570 (17)	0.4233 (18)
Lolo Creek	17	49	43.58	1.12243 (4)	0.6826 (14)
Upper Clark Fork 2	12	7	25.62	0.2733 (25)	0.2217 (25)
Upper Clark Fork 4	9	32	21.23	1.5071 (2)	0.8455 (6)
Lower Blackfoot	8	10	14.08	0.7101 (13)	0.7309 (12)
Warm Springs Creek	10	14	7.48	1.8708 (1)	0.7707 (9)
Silver Bow Creek	0	0	0.0	0.0 (26)	0.0 (26)

TABLE II

Estimated Parameters on Catch and Travel Cost*

Model	Catch parameter	Travel-cost parameter
Catch rates and parameter jointly estimated	0.126 + 0.02604 (skill level) (1.581) (1.633)	−0.00722 (3.739)
Nonjoint (sequential) estimation using the simple Poisson average catch rates	−0.030 + 0.01601 (skill level) (0.572) (1.522)	−0.006507 (3.845)

* For resident anglers. Numbers in parentheses are t statistics.

of camping. Many angler characteristics were also included. The likelihood under this stochastic assumption for catch is

$$\log L\big(\mathbf{Z}|\boldsymbol{\beta}, c_1^*, \ldots, c_n^*, \sigma_\epsilon^2\big) = \sum_j \bigg(\sum_i y_{ji} \log \Pr\big(y_{ji} = 1\big) + \sum_l \log \Pr\big(\eta_{jl}\big)\bigg), \quad (33)$$

where the first probability is from (29) and (30), and the second probability is from (32). The jointly estimated catch rates are reported in column 5 of Table I.[15] Note that columns 4 and 5 are quite similar (with a correlation of 0.79), except at those sites with few observed hours of fishing. For example, the simple average for Upper Clark Fork is 1.5 fish/hour, based on 21 hours of reported catch from nine trips, but the jointly estimated expected catch rate is 0.85. In contrast, Beaverhead, with 464.4 hours of observed catch from 172 trips, has a simple Poisson average catch of 0.6331 and a jointly estimated expected catch of 0.6366.

The estimated coefficients on catch and travel cost for resident anglers are reported in row 1 of Table II. Note that the parameter on expected catch rate is a function of the skill level of the angler. Row 2 reports the same parameters for conventional estimation (when the simple Poisson average catch rate is used as a proxy).[16] For both simultaneous and conventional estimation, likelihood ratio tests indicate that travel cost and expected catch are significant determinants of choice. The two travel-cost parameters are very similar, but the estimated impact of catch when estimated conventionally has a significant downward bias.

6. SUMMARY

Econometric theory tells us that using the average of the observed catch rates at a site as a proxy for the expected catch rate at a site will make catch appear less important than it is as a determinant of participation and site choice. An empirical example shows that the downward bias can be substantial. The existing literature on recreational fishing demand has therefore underestimated the impact of catch

[15]To anchor the expected catch rates, the expected catch rate for Madison 2 was fixed at its simple Poisson average, 0.7260.

[16]Note that, when conventionally estimated, the catch parameter has the wrong sign for anglers with very low skill levels (level 1 only; the range on skill was 1 to 7, the average for residents in the sample was 4.86). Seven anglers have a skill level of 1.

rates on participation and site choice. Consistent and efficient estimates of expected catch rates and the parameters in the demand functions can be obtained by simultaneously estimating them in a maximum likelihood framework. As demonstrated, empirical implementation is straightforward for simple trip demand models, discrete choice models of participation, and repeated nested logit models of participation and site choice.

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