Environmental Quality Preference and Benefit Estimation in Multinomial Probit Models: A Simulation Approach

Heng Z. Chen and Stephen R. Cosslett

Simulated maximum likelihood is used to estimate a random parameter multinomial probit model of destination choice for recreational fishing trips, formulated to accommodate varying tastes and varying perceptions of environmental quality across individuals. The restricted likelihood ratio test strongly rejects the independent probit model, which is similar to the independent logit model in both the parameter and benefit estimates. Furthermore, both the Krinsky-Robb and bootstrapping procedures suggest that the benefit (standard deviation) of an environmental policy is found to be markedly lower (higher) when heterogeneous preferences are taken into account.

Key words: benefit of environmental site quality change, bootstrapping procedure, Krinsky-Robb procedure, random parameter and constant parameter multinomial probit models, recreational fishing, simulated maximum likelihood estimation.

The random utility model of destination choice is often specified as $U_j^i = p_j^i \alpha + x_j \beta + \epsilon_j^i$, where p_j^i is the trip cost to site j for individual i, x_j is a vector of attributes of site j, and α and β are constant preference parameters. The stochastic term ϵ_j^i allows for idiosyncratic taste variation across individuals, which is not observed by the researcher. If ϵ_j^i follows the type I extreme value (EV) distribution or a generalized EV distribution, the model becomes an independent or nested multinomial logit, respectively. Logit models have been widely used to study destination (or site) choice in recreational fishing under the hypothesis of random utility maximization (see Morey, Rowe, and Watson, and Parsons and Kealy, among others). These models can be used to estimate the benefits of improvements in the environmental quality of the site as represented by components of x_j (Small and Rosen, Hanemann).

Logit models, however, have some undesirable properties. The well-known independence of irrelevant alternatives property (IIA) of the independent logit model restricts the pattern of substitutability across alternatives and makes the model less likely to reflect reality. Although the nested logit model relaxes the IIA assumption, the choice of nest structure can be ad hoc and the pattern of correlations admitted by the model is limited because the IIA assumption is still maintained within each nest.

More importantly for recreational site choice models, the assumption of constant parameters β implies the same marginal utility of site quality for all individuals. To reflect varying marginal utility, or varying tastes, for site quality, one can specify a random (or varying) parameter model with $\beta^i = \beta + \delta^i$, where β is the average taste and δ^i represents individual-specific taste variations. β^i or δ^i can be viewed as realizations of a random variable.[1] Heterogenous tastes for site quality can thus be accommodated as $U_j^i = p_j^i \alpha + x_j \beta^i + \epsilon_j^i = p_j^i \alpha + x_j \beta + x_j \delta^i$

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[1] Although the trip cost parameter can also be specified as a random variable α^i, we think that parameter variation is likely to be less important for α than for the coefficients of site quality x_j because the trip cost p_j^i is already individual specific as well as site specific. Furthermore, because the utility is ordinal, we can normalize the model $U_j^i = p_j^i \alpha^i + x_j \beta^i + \epsilon_j^i$ by α/α^i: $U_j^{*i} = p_j^i \alpha + x_j(\alpha/\alpha^i)\beta^i + (\alpha/\alpha^i)\epsilon_j^i$, where α is a constant that normalizes the variance of $(\alpha/\alpha^i)\epsilon_j^i$. The model is then specified by the distributions of ϵ_j^i/α^i and δ^i/α^i. Assuming normality for utility differences expressed in monetary terms seems no less plausible than for utilities expressed in some other units of internal satisfaction.

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$+ \epsilon_j^i$.[2] Econometrically, this is most easily implemented as a random (or varying) parameter multinomial probit (VPMNP) model, by assuming that δ and ϵ are independent and that each of them follows a multivariate normal distribution.

Although Hausman and Wise investigated transportation mode choice using the random parameter specification, their study was limited to investigating three alternatives. The limited computing power and econometric estimation methods available at that time made it impractical to evaluate choice probabilities for the multinomial probit model with more than four choice alternatives. For recreational fishing and other site-choice models, however, the number of feasible sites can be very large. Different individuals can have different feasible sites as well. For example, there can be as many as thirty-seven feasible sites for some individuals in our study. It is therefore interesting to examine whether there exist any differences between the widely used independent multinomial logit model (FPMNL), the independent multinomial probit model (FPMNP), and the VPMNP with the correlation structure induced by $x\delta$ and possibly also by a nondiagonal covariance matrix for ϵ. Such comparisons were not possible until recent advances in simulation methods by McFadden, by Pakes and Pollard, and others.

Because the variance of δ in the VPMNP model is non-negative, testing the significance of the varying parameter specification using, for example, the likelihood ratio (LR)-statistic should take account of the non-negativity restrictions. Simply applying the conventional unrestricted LR-statistic will bias the test (see Bartholomew or Barlow et al., among others). In this paper, we develop a procedure to simulate the LR-statistic for testing the varying parameter specification in the VPMNP model under the non-negativity restrictions.

In addition, when the site-choice model is applied to recreational fishing, we are interested in estimating the benefits of a change in environmental quality at some of the sites. To this end, we have to estimate the expected maximum utility from the model before and after the policy implementation. In the case of logit models, this involves estimation of the inclusive values. However, as with the choice probabilities, there is no computationally tractable expression for the expected maximum utility of probit models. In this paper, therefore, we use an unbiased frequency simulator to estimate the expected maximum utility, and thus the mean benefit due to a policy change, in multinomial probit models. Furthermore, we use the Krinsky–Robb procedure (Krinsky and Robb) to estimate the distribution of the mean benefit for the logit and probit models considered in this paper. As a comparison to the Krinsky–Robb procedure, we also bootstrap the distribution of the mean benefit.

Using 1983–84 survey data on Michigan anglers' recreational fishing, we estimated the FPMNL, MPMNP, and VPMNP models with the trip cost and three other site quality indices as the explanatory variables. The indices are the salmon catch rate, forest coverage in percentage, and a dummy variable to indicate whether the site is contaminated. More detailed descriptions of these variables are given below. The mean benefits and their distribution were also estimated for cleaning up the contamination represented by the dummy variable.

In the rest of this article, after the VPMNP model is formulated, we briefly illustrate simulated maximum likelihood estimation (MLE) using the smooth recursive normal simulator, known as the GHK sampling method[3] [see Hajivassiliou or Keane (1993, 1994) for an exposition of the method]. The restricted LR-statistic for testing the varying parameter specification under the non-negativity restrictions is also discussed. For benefit estimation in the probit models, the expected maximum utility is then simulated using the unbiased frequency simulator. Following a description of the survey data, we present the results of three models that are compared to assess the implications of different distribution assumptions with and without varying tastes for site quality. The benefit due to a policy change is also estimated using the three models and the results

[2] The model can also be formulated in terms of a varying perceptions model. Suppose that perceived site quality z_j^i is related to measured site quality x_j by $z_j^i = x_j(1 + \eta^i) + \zeta_j^i$, where the random terms η^i and ζ_j^i allow for both systematic and site-dependent perception variation for individual i. In general, the stochastic component of U_j^i then has the same form as in the "varying tastes" model, $x_j\delta^i + \epsilon_j^{*i}$, where $\delta^i \equiv \eta^i\beta$ and $\epsilon_j^{*i} \equiv \epsilon_j^i + \zeta_j^i\beta$. Note that the empirical model estimated later in this paper is based on a simplified covariance structure: if interpreted as a varying perceptions model, it corresponds to the special case where the site-dependent terms ζ_j^i are independent.

[3] The acronym GHK refers to its authors: Geweke; Hajivassiliou and McFadden; and Keane.

are compared. We conclude the paper with some final remarks.

Simulated MLE, Restricted LR-Test, and Benefit Estimation

Simulated MLE of Multinomial Probit Models

To model heterogeneous preferences for site quality across individuals, a random parameter model can be specified as

$$(1) \quad U_j^i = p_j^i\alpha + x_j\beta^i + \varepsilon_j^i = p_j^i\alpha + x_j\beta + u_j^i$$

where the stochastic term is given by $u_j^i = x_j\delta^i + \epsilon_j^i$. If $\delta_k^i = 0$, the random parameter specification for the kth site quality variable is not informative.[4] If this holds for all $k = 1, \ldots, K$, the model degenerates to the conventional constant parameter model.

To estimate the parameters in equation (1), we assume that δ and ϵ are independent, and each of them follows a multivariate normal distribution with $\epsilon \propto N(0, \Sigma_\epsilon)$ and $\delta \propto N(0, \Sigma_\delta)$. When $\Sigma_\delta = \text{diag}(\sigma_{\delta 1}^2, \ldots, \sigma_{\delta K}^2)$, the covariance matrix for u is given by $\Sigma_u = \Sigma_\epsilon + \Sigma_{x\delta}$, where

$$\Sigma_\epsilon = \begin{pmatrix} \sigma_{\epsilon 11}^2 & \cdots & \sigma_{\epsilon 1J}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{\epsilon J1}^2 & \cdots & \sigma_{\epsilon JJ}^2 \end{pmatrix}$$

and

$$\Sigma_{x\delta} = \begin{pmatrix} \sum_k \sigma_{\delta k}^2 x_{1k}^2 & \cdots & \sum_k \sigma_{\delta k}^2 x_{1k}x_{Jk} \\ \vdots & \ddots & \vdots \\ \sum_k \sigma_{\delta k}^2 x_{1k}x_{Jk} & \cdots & \sum_k \sigma_{\delta k}^2 x_{Jk}^2 \end{pmatrix}.$$

It is clear that if $\delta_k \neq 0$ for any k, the off-diagonal elements of the covariance matrix Σ_u are nonzero. Thus, correlations across alternatives can be introduced in two ways. One is due to varying tastes for site quality with $\Sigma_k\sigma_{\delta k}^2 x_{jk}x_{j'k} \neq 0$ for $j \neq j'$. The other is due to the correlations among components of ϵ with $\sigma_{\epsilon_{jj'}}^2 \neq 0$ for $j \neq j'$.

Let j be the chosen alternative from the J feasible alternatives for a given individual. The choice probability $p(j)$ for the multinomial probit model is then:

$$p(j) = \int_{-\infty}^{\infty} du_j \int_{-\infty}^{(p_j-p_1)\alpha+(x_j-x_1)\beta+u_j} du_1 \cdots \int_{-\infty}^{(p_j-p_J)\alpha+(x_j-x_J)\beta+u_j} du_J \cdot f(u)$$

where $f(u)$ is the multivariate normal density function for u with mean 0 and covariance Σ_u. Unlike the multinomial logit model in which $p(j)$ can be expressed as a ratio of exponential functions, $p(j)$ for the multinomial probit model is difficult to evaluate because it involves high-dimensional integration. To overcome this difficulty, a number of simulators have been introduced to approximate the choice probabilities through Monte Carlo simulations, including the frequency method (Lerman and Manski), the importance sampling method (McFadden), Stern's method (Stern), and, more recently, the smooth recursive sampling GHK simulator.

The GHK simulator, which we apply in this paper, has several advantages. First, the resulting simulated probabilities are continuous in the parameter space, and therefore estimation can be performed by using standard optimization packages. Furthermore, Börsch-Supan and Hajivassiliou show that the GHK simulator is unbiased for any given number of replications R, and it generates substantially smaller variance than the frequency simulator and Stern's simulator. Based on the root-mean-square error criterion, Hajivassiliou, McFadden, and Ruud show that the GHK simulator is unambiguously the most reliable method for simulating normal probabilities, compared to twelve other simulators considered.

To estimate the parameters by the simulated MLE method, we need only replace the choice probabilities in the likelihood function by the simulated probabilities using the GHK simulator. Details of the computational steps required to simulate the probabilities can be found in Börsch-Supan and Hajivassiliou. As the sample size and the simulation replications increase, maximization of the simulated likelihood yields parameter estimates that possess the asymptotic properties of conventional

[4] To simplify the notation, we omit the superscript i in the remainder of this section. Also notice that the dimension of the random vectors (ϵ and u), of the covariance matrices (Σ_ϵ and Σ_u), and the total number of feasible alternatives (J) should also be interpreted as individual specific.

MLEs (see Gourieroux and Monfort).[5] Statistical inference for the simulated MLE can also be implemented.

Restricted Likelihood Ratio Test

In testing the null hypothesis of a fixed parameter model against the alternative of a varying parameter model, i.e.,

- H_0: $\sigma^2_{\delta 1} = \sigma^2_{\delta 2} = \cdots = \sigma^2_{\delta K} = 0$
- H_1: $\sigma^2_{\delta k} \geq 0 \quad (k = 1, \ldots, K)$

  with $\sigma^2_{\delta k} > 0$ for some k

where $\sigma^2_{\delta k} = \text{var}(\delta_k)$, we must take account of the inequality constraints in the alternative. This is because the covariance matrix $\Sigma_u = \Sigma_\epsilon + \Sigma_{x\delta}$ can still be definite positive for small negative values of $\sigma^2_{\delta k}$, so the condition $\sigma^2_{\delta k} \geq 0$ is not automatically imposed by requiring the log likelihood to be well defined. Instead, the inequality constraints have to be imposed as additional restrictions on the maximum likelihood estimator. Under H_0, following Bartholomew or Barlow et al., the log LR-statistic for testing H_0 against H_1 is asymptotically a mixture of χ^2 distributions

$$2[\log L(\tilde{\theta}) - \log L(\tilde{\theta}_0)] \propto \sum_{q=0}^{K} \pi_q \chi^2(q)$$

where $\tilde{\theta}$ and $\tilde{\theta}_0$ denote the restricted MLEs of $\sigma^2_{\delta k}$ for $k = 1, \ldots, K$ subject to H_1 and H_0, respectively. $\chi^2(q)$ denotes a random variable with a chi-squared distribution with q degrees of freedom, and $\chi^2(0)$ denotes a degenerate random variable always equal to zero. π_q is the probability that q of the K elements in $\tilde{\theta}$ are strictly positive for $q = 0, 1, \ldots, K$. This test statistic can be viewed as a multidimensional generalization of the conventional one-sided t-test for, say, H_0:$\sigma^2_{\delta 1} = 0$ against H_1: $\sigma^2_{\delta 1} > 0$ in the case $K = 1$. Failure to account for the non-negativity restrictions of the alternative hypothesis will bias toward acceptance of the null hypothesis. The detailed steps for simulating the asymptotic distribution of the LR-statistic are available from the authors upon request.

Expected Maximum Utility

The expected maximum utility $\bar{U}_m$ of making a choice from the J feasible alternatives is

$$\text{(2)} \quad \bar{U}_m = \int_{-\infty}^{\infty} \max_j \, (p_j\alpha + x_j\beta + u_j) f(u)\, du$$
$$= \sum_{j=1}^{J} \int_{-\infty}^{\infty} (p_j\alpha + x_j\beta + u_j) \times \text{I}[p_j\alpha + x_j\beta + u_j \geq p_l\alpha + x_l\beta + u_l, \forall l] f(u)\, du$$

where the indicator $\text{I}[A] = 1$, if A is true, 0 otherwise. For example, when u follows the type I EV distribution in a logit model, the expected maximum utility (or the inclusive value) $\bar{U}_m$ has a closed-form solution $\ln[\Sigma_{j=1}^{J} \exp(x_j\beta)] + \gamma$, where $\gamma = 0.577 \cdots$ is Euler's constant. However, when u follows a normal distribution there is no closed-form expression for the expected maximum utility $\bar{U}_m$. In this paper, the expected maximum $\bar{U}_m$ is estimated by drawing the random terms u_j with $j = 1, \ldots, J$, and finding the maximum utility across sites for each replication r. The average of the maximum over replications

$$\hat{\bar{U}}_m = \frac{1}{R} \sum_{r=1}^{R} \sum_{j=1}^{J} (p_j\alpha + x_j\beta + u_j^r) \times \text{I}[p_j\alpha + x_j\beta + u_j^r \geq p_l\alpha + x_l\beta + u_l^r, \forall l]$$

is unbiased because $E(\hat{\bar{U}}_m x) = \bar{U}_m$.

The Mean Benefit and Its Distribution

In recreational demand studies, one important objective of estimating a random utility model is to estimate the benefit for a measure of proposed environmental site quality change. If the environmental quality at one or more sites is to be changed such that the array x of site attributes changes from x^0 to x^1, and if the marginal utility of money $-\alpha$ remains constant, the mean benefit can be estimated by

$$\text{(3)} \quad EW(x^1 | x^0) = \frac{\bar{U}_m(x^1) - \bar{U}_m(x^0)}{-\alpha}.$$

The numerator measures the change in expected maximum utility due to the policy implementation, and the denominator converts

[5] Other estimation methods such as the method of simulated moments (MSM), or the method of simulated scores (MSS) can also be used. Each of them shares some advantages and disadvantages that are not our focus here. For a review, see Gourieroux and Monfort, or Hajivassiliou, McFadden, and Ruud.

the utility difference into dollar units. See Small and Rosen or Hanemann for a discussion.

Furthermore, for policy purposes, it is also desirable to obtain the distribution of the mean benefit estimate, using either the Krinsky–Robb procedure or the bootstrapping procedure. For the Krinsky–Robb procedure, we draw S times from the asymptotic normal distribution of the parameter estimates, and then calculate the benefit equation (3) for each of the draws. On the other hand, the bootstrapping method constructs S new data sets, using the parameter point estimates and the explanatory variables, by generating error terms u and comparing the constructed utility $\alpha + \beta x + u$ across alternatives to create the polychotomous dependent variable y. The constructed data sets $\{x,y\}$ are then used to compute S new parameter estimates. As a result, the mean benefit equation (3) and its distribution can be estimated using the estimated new parameters.

While the distribution of the mean benefit can easily be obtained for the logit model with the above procedures using either the Krinsky–Robb or bootstrapping procedure, some further steps are required in estimating the distribution for the probit model because it does not have a closed-form expression for $\bar{U}_m$. That is, we have to first obtain $\bar{U}_m$ for a given α^s, β^s, and Σ_δ^s before the distribution of the mean benefit can be derived. Specifically, (*a*) for a given α^s, β^s, and Σ_δ^s, we obtain draws u^r for $r = 1, \ldots, R$. (*b*) The maximum utility difference after and before the policy implementation is calculated for $r = 1, \ldots, R$. The mean benefit in equation (3) is the average of the difference over R replications divided by $-\alpha^s$. (*c*) The distribution of the mean benefit is obtained by repeating steps (*a*) and (*b*) for $s = 1, \ldots, S$. In the next section, we will use a policy scenario to illustrate the similarity and difference in the distribution across different models using both the Krinsky–Robb and bootstrapping procedures.

Data and Estimation Results

We use a subset of the data on recreational fishing reported by Jones and Sung. A brief discussion of the data is given here; see Jones and Sung for further details. The data set consists of two parts. One is from a 1983–84 survey of Michigan anglers by the Michigan Department of Natural Resources. The survey questionnaires were mailed out throughout the fishing season, asking about the most recent fishing trip. From the returned questionnaires, 338 single-day fishing trips that target salmon species in the Great Lakes (Michigan, Superior, Huron, and Erie) were selected for this study. The elementary site was defined as each county. There were a total of 41 Great Lakes sites in Michigan that supported salmon fishing. The feasible set for each individual consists of all the sites that are within the maximum driving distance observed in the survey data set. For every feasible site j, the trip cost variable $cost_j^i$ is the round-trip driving distance between each individual's home site and the feasible site multiplied by the American Automobile Association mileage cost at \$0.28 per mile. Thus, $cost_j^i$ is individual and site specific. Table 1 reports summary statistics for the trip cost.

Table 1. Data Summary Statistics

Variables	Mean	Standard Deviation	Minimum	Maximum	Sample Size
cost	11.140	13.370	0.9	75.40	338
aoc	0.341	0.480	0.0	1.00	41
forest	0.541	0.291	0.7	0.97	41
salmon	0.036	0.044	0.0	0.22	287

The other part of the data set consists of environmental quality variables provided by the Michigan Department of Natural Resources. Summary statistics for the three site quality variables used in this article can also be found in table 1. Specifically, aoc_j is a dummy variable with value of 1 to indicate that site j is designated as an area of concern for toxic contamination by the International Joint Commission; 0, otherwise. Note that this index is intended to reflect only qualitatively the level of contaminants such as mercury, polychlorinated biphenyls, and/or dioxin contained in the fish caught at the site.[6] There is no site restriction and individuals can still access and fish at site j even if $aoc_j = 1$. The variable $forest_j$ is the percentage of the forest coverage at site j; $salmon_{jt}$ is the number of salmon caught per hour. The variable is site j and month t specific because the salmon catch rate in the Great Lakes can change significantly over time in the open water fishing

[6] For details, see Michigan Fish Contaminant Monitoring Program, and Michigan Fishing Guide, 1983 through 1996.

Table 2. FPMNL Model (Log Likelihood Value is −523.21)

Variables	Parameter Estimates	t-Statistics	Normalized Estimates[a]
α (cost/100)	−17.300	−16.547	−1.000
β_1 (aoc)	−1.583	−8.916	−0.092
β_2 (forest)	2.532	4.775	0.146
β_3 (salmon)	7.407	3.542	0.428

[a] Normalized estimates are parameter estimates divided by $-\alpha$.

Table 3. FPMNP Model (Log Likelihood Value is −539.89)

Variables	Parameter Estimates	t-Statistics	Normalized Estimates[a]
α (cost/100)	−9.122	−15.096	−1.000
β_1 (aoc)	−0.981	−8.656	−0.108
β_2 (forest)	1.556	4.639	0.171
β_3 (salmon)	4.564	3.144	0.500

[a] Normalized estimates are parameter estimates divided by $-\alpha$.

season (from April to October) (for details, see Jones and Sung).

Using the trip cost and the three site quality variables, we specify the utility of a fishing trip to site j in month t as

$$\begin{aligned} U_{jt}^{i} &= cost_j^i\alpha + aoc_j\beta_1^i + forest_j\beta_2^i \\ &\quad + salmon_{jt}\beta_3^i + \epsilon_j^i \\ &= cost_j^i\alpha + aoc_j\beta_1 + forest_j\beta_2 \\ &\quad + salmon_{jt}\beta_3 + u_j^i \end{aligned}$$

where the stochastic term is $u_j^i = aoc_j\delta_1^i + forest_j\delta_2^i + salmon_{jt}\delta_3^i + \epsilon_j^i$. Three models are estimated to examine the implications of different distribution assumptions for ϵ and the importance of the varying taste specification δ.

The first model (FPMNL) uses a type I EV distribution for ϵ in which the ϵ_js are independent with $\delta^i = 0$. It is estimated by conventional maximum likelihood. Table 2 presents the parameter estimates and the t-statistics. The log likelihood value is −523.21.

The second model (FPMNP) maintains the independence assumption for the ϵ_js, but the type I EV distribution is replaced by a standard normal distribution with Σ_ϵ = diag(1, . . . , 1). As in the FPMNL, there is no taste variation: $\delta^i = 0$. Parameter estimates obtained by simulated MLE with 2,000 replications are reported in table 3. The log likelihood value decreases from −523.21 for the FPMNL to −539.89 for the FPMNP as a result of changing the distribution assumption for ϵ. This indicates that the EV distribution fits the data set better than the normal distribution, provided that the error terms are truly independent. By comparing the normalized parameter estimates or the ratios β_k/α for $k = 1$, 2, 3 for the FPMNL and FPMNP models, one can see that the estimated logit and probit model are quite similar under the independence assumption.

As the sample size and the number of replications increase, the parameter estimates of the simulated MLE possess the same asymptotic properties as those of the MLE. However, there are no guidelines as to how many replications are empirically needed for our given problem. In order to verify that 2,000 replications is appropriate, the FPMNP model is also estimated with different replications (R = 10, 50, . . . , 2,000). As the replication increases, the log likelihood value gradually increases and stabilizes around −540 after the replication is larger than 100. The parameter estimates also stabilize. It is noticed that when the replication is small, such as 10, the simulation noise can be quite severe.

The third model we estimated is the VPMNP model in which the covariance matrix is given by $\Sigma_\epsilon + \Sigma_{x\delta}$, where Σ_ϵ = diag(1, . . . , 1).[7] To compare the FPMNP model with the VPMNP model, we assume that the correlation across alternatives is caused only by $x\delta$ due to heterogeneous preferences for aoc, forst, and salmon. Table 4 reports the parameter estimates for the VPMNP model with 2,000 replications.

With the same 2,000 replications, the log likelihood value of the VPMNP is −473.88, as compared with −539.89 for the FPMNP, yielding the LR-statistic $2[\log L(\tilde\theta) - \log L(\tilde\theta_0)] = 132.02$ between the two models. Because the asymptotic distribution of the restricted LR-statistic under the inequality restrictions is a mixture of χ^2 distributions, we simulate this distribution and find that the critical value of the statistic with degrees of freedom three or lower is 8.73 at the 1% signif-

[7] A more general model with $\sigma^2_{\epsilon jj'} \neq 0$ for $j \neq j'$ could be estimated if we had enough data observations to recover the identifiable parameters in the covariance matrix Σ_ϵ.

Table 4. VPMNP Model (Log Likelihood Value is −473.88)

Variables	Parameter Estimates	t-Statistics	Normalized Estimates[a]
α (cost/100)	−16.280	−14.015	−1.000
β_1 (aoc)	−3.243	−4.274	−0.199
β_2 (forest)	1.437	2.422	0.088
β_3 (salmon)	6.474	3.175	0.398
$\sigma_{\delta 1}$ (aoc)	4.354	4.219	0.267
$\sigma_{\delta 2}$ (forest)	4.547	5.685	0.279
$\delta_{\delta 3}$ (salmon)	0.009	0.002	0.000

[a] Normalized estimates are parameter estimates divided by $-\alpha$.

icance level (or 5.49 at the 5% significance level). Thus, the FPMNP model is strongly rejected at the 1% level. This indicates that the independent FPMNP model does not adequately describe the data variations, and thus its parameter estimates are likely to be inconsistent because the independence assumption across alternatives cannot hold.

For the VPMNP model, the estimates of α, β_1, β_2, and β_3 are significant: the trip cost and the three site quality variables are useful in determining an individual's site choices. As before, we divide the estimated marginal utility of site quality β by the estimated marginal utility of money ($-\alpha$) in order to compare results from models with different normalizations for utility. It can be seen that the marginal values of site quality (in dollar units) differ between the correlated VPMNP model on the one hand and the FPMNL and FPMNP models on the other hand (table 4). For example, the estimate of $-\beta_1/\alpha$ in the VPMNP model is twice what it is in the FPMNP or the FPMNL model.

We further estimate and compare the mean benefit and its distribution due to a change in site quality in each of the three estimated models. The policy scenario considered here is to clean up the environmental toxic contamination to the extent necessary to reclassify the fourteen contaminated Great Lakes sites in Michigan from aoc = 1 to aoc = 0. Using equation (3), the Krinsky–Robb procedure with S = 1,000 replications yields the mean benefit of the clean-up of $3.06 per trip with a standard deviation of 0.22 for the FPMNL model (see table 5). For both probit models, we use R = 1,000 and S = 1,000. The mean benefit for the FPMNP and VPMNP models is $3.42 and $0.73 per trip with standard deviations of 0.22 and 0.84, respectively.

Table 5. Per Trip Benefit

Models	Krinsky–Robb Procedure EW	Krinsky–Robb Procedure Standard Deviation	Bootstrapping Procedure EW	Bootstrapping Procedure Standard Deviation
FPMNL	3.06	0.22	3.08	0.25
FPMNP	3.42	0.22	3.43	0.24
VPMNP	0.73	0.84	0.70	0.85

On the other hand, the bootstrapping procedure suggests that we should reestimate the model's parameters by constructing new data sets. For the logit model, we created 500 data sets, which led to 500 different estimates of α and β. As a result, equation (3) yields 500 benefit estimates with a mean of $3.08 per trip and a standard deviation of 0.25. This is similar to the estimates from the Krinsky–Robb procedure. For each of the two probit models, we also created 500 data sets based on the point estimates of the model's parameters and their error distributions. To estimate 500 different sets of parameters using the GHK simulation method for each probit model, 100 replications are used because it is indicated that the likelihood values have already stabilized. As is the case of the logit model, equation (3) is used to calculate 500 different benefits for both probit models. The means (standard deviations) for the FPMNP model and the VPMNP model are $3.43 (0.24) and $0.70 (0.85), respectively. These results are similar to those obtained using the Krinsky–Robb procedure.

Final Remarks

Comparing the FPMNP model and the VPMNP model, one can see that the varying parameter specification greatly improves the model's goodness of fit. This specification can be important because in many cases the explanatory variables, such as site quality, are measured by a set of technical numbers that do not vary across individuals. The concern is whether the site quality indices combined with constant preference parameters can adequately accommodate individual tastes of the site quality. The estimation results suggest that the varying parameter specification provides a significant improvement over the constant parameter specification. The restricted LR-test strongly rejects the fixed parameter specification.

In many empirical situations, we are also interested in assessing policy benefits. By using the frequency simulator, it is shown that the mean benefit estimates for removing the contamination represented by aoc are similar in the FPMNL and FPMNP models, a finding that resembles the similarity in the parameter estimates between the two models. However, the estimated mean benefit using the correlated VPMNP model is markedly lower than the estimated means from the FPMNP and FPMNL models. Furthermore, both the Krinsky–Robb and bootstrapping procedures suggest that standard deviations of the benefits for the VPMNP model are also higher than those for the FPMNL and FPMNP models. This indicates that to obtain a benefit estimate with good precision within the VPMNP model framework, the sample size of 338 may not be large enough because the precision of the benefit estimate is a function of the precision of the model's parameter estimates.

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